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Improving the Efficiency of Population Mean Using Double Sampling Technique in the Presence of Nonresponse

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Abstract

This paper proposes some classical estimators of the population mean using the data on two auxiliary variables under a two-phase sampling scheme with subsampling the nonrespondents. This study considers the scenario in which nonresponse affects the study and first auxiliary variables but not the supporting auxiliary variable. The expression for the bias and means square error of the proposed estimator has been derived up to the first order of approximation. Additionally, an efficiency comparison between the proposed estimator and some well-known existing estimators of the population mean has been conducted. Furthermore, certain empirical studies have corroborated the theoretical derivations, and the findings indicate that employing two auxiliary variables in estimating the population parameters in the presence of nonresponse is more profitable in terms of mean square error reduction.

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Introduction

In some practical situation, if the information on more than one auxiliary variable is easily available; the information on all such variables could be utilized to provide more precised estimators of the population parameter. Olkin (1958) was the first to propose the multivariate ratio estimators of the population mean utilizing the information on a number of auxiliary variables. It was further extended by Srivastava (1966) ^[28], Rao and Mudhoklar (1967), Singh (1967) and Sahai (1980) ^[24]. Moreover, Chand (1975), Kirgyera (1980), Kadilar and Cingi (2005) ^[14] and Iseh and Enang (2021) ^[8, 9, 10] made useful contributions in this area. Shukla and Thakur (2012) ^[25], Kumar and Sharma (2020) ^[19], and others have suggested a variety of estimators of the population mean using the information on two auxiliary variables.

In sample survey, it is of great importance to deal with the problem of nonresponse. When the investigator fails to get information from the expected units of the population, nonresponse occurred. If the characteristics of the nonresponding units are very similar to those of responding units, then the nonresponse does not affect the outcome of the estimates adversely. However, practically, it is always seen that the responding units are not very similar to the nonresponding units and hence, nonresponse becomes a challenge in estimation under such situations. The problem of subsampling the nonrespondents was first suggested by Hansen and Hurwitz (1946) ^[7] to reduce the effect of nonresponse error in sample survey. They obtained an unbiased estimator with the technique of subsampling the nonrespondents, while the effect of the nonresponse was only noticed in the variance. Sarndal, Swensson and Wretman (1992) suggested the technique of estimating the nonresponse propensity. Iseh and Bassey (2021a) and Iseh and Bassey (2021b) extended Sarndal, Swensson and Wretman (1992) to calibration technique with some outstanding results. However, there are several authors who have dealt extensively with the problem of nonresponse either by proposing various estimators of the population parameters or by extending the Hansen and Hurwitz (1946) ^[7] technique of subsampling the nonresponse. Some of which include Olkin (1986) ^[22], Khare and Srivastava (1993) ^[16], Khare and Srivastava (1995) ^[17], Okafor and Lee (2000) ^[20], Singh and Kumar (2007, 2009) ^[26, 27], Chaudhary and Kumar (2016) ^[5], Anieting and Enang (2020) ^[2], Iseh and Bassey (2022, 2024), Ikot and Iseh (2024) ^[12, 13] and Bassey and Iseh (2025) ^[4]. In addition, Tabasum and Khan (2004), Pal and Singh (2018), Khare and Sinhan (2019) ^[15], etc, are among several authors who have discussed the problem of nonresponse

in two-phase sampling scheme. The problem of estimating the parameter in two-phase sampling would be more complex when the nonresponse is observed on both study and auxiliary variables. The problem of such situation was handled by Chaudhary, Kumar, and Vishwakarma (2021) [6]. To get the improved estimators of the population mean under the present situation, we seek to utilize information on two auxiliary variables. The proposed estimators are formulated under the assumption that the additional (second) auxiliary variable is free from nonresponse. It is further assumed that the first auxiliary variable has a high degree of correlation with the study variable than the second auxiliary variable. In order to validate the theoretical findings, two different sets of empirical data have been considered.

Sampling Strategy

Supposed a population comprises of N units ($U_1, U_2, \dots, U_N$). Let Y be the study variable with population mean $\bar{Y}$. Let X and Z be the two auxiliary variables with respective population means $\bar{X}$ and $\bar{Z}$. Let y_i, x_i and z_i ($i = 1, 2, \dots, N$) be observations on the i^{th} unit in the population for the variables, Y, X , and Z respectively. Assuming that nonresponse is observed on the study variable Y and auxiliary variable X whereas the auxiliary variable Z is free from the non-response. It is further assumed that the population mean $\bar{X}$ of the auxiliary variable X is unknown. In the present situation, we first estimate $\bar{X}$ adopting the concept of double sampling scheme and then use the estimate so obtained along with the information on the other auxiliary variable Z to get the estimate of $\bar{X}$. Let a larger n' units be selected from N units using simple random sampling without replacement (SRSWOR) scheme at the first phase and then a smaller subsample of n units be selected from n' ($n < n'$) units using SRSWOR scheme at the second phase. At the first phase, out of n' units, n'_1 units respond and n'_2 units do not respond on the auxiliary variable X . Now a subsample of h'_2 unit is selected from the n'_2 units using SRSWOR scheme ($h'_2 = n'_2 / L', L' > 1$) and the information from all the h'_2 is collected [see Hansen and Hurwitz (1946)]. Thus, the estimators of $\bar{X}$ and $\bar{Z}$ at the first phase are respectively given as

$$\bar{x}'^* = \frac{n'_1 \bar{x}'_{n'_1} + n'_2 \bar{x}'_{h'_2}}{n'}$$

And

$$\bar{z}' = \frac{1}{n'} \sum_{i=1}^{n'} z_i$$

where $\bar{x}'_{n'_1}$ and $\bar{x}'_{h'_2}$ are respectively the means based on n'_1 responding units and h'_2 non-responding units for the auxiliary variable X .

The variance of the estimators $\bar{x}'^*$ and $\bar{z}'$ are respectively given by

$$V(\bar{x}'^*) = \left(\frac{1}{n'} - \frac{1}{N} \right) S_X^2 + \frac{(L'-1)}{n} W_2 S_{X_2}^2$$

$$V(\bar{z}') = \left(\frac{1}{n} - \frac{1}{N} \right) S_Z^2$$

where S_X^2 and $S_{X_2}^2$ are respectively the population mean squares of entire group and non-response of the auxiliary variable X . S_Z^2 is the population mean square of the auxiliary variable Z and W_2 represents the non-response rate in the population.

Furthermore, at the second phase, it is observed that there are n_1 responding units and n_2 non-responding units in the sample of n units for study variable Y and auxiliary variable X , while the auxiliary variable Z is free from the nonresponse. Adopting the Hansen and Hurwitz (1946) [7] technique of sub-sampling the non-respondents, we select a sub-sample of h_2 unit from n_2 non-responding unit using SRSWOR scheme at the second phase ($h_2 = n_2 / L', L' > 1$) and gather the information from all the h_2 units. Thus, the Hansen and Hurwitz (1946) [7] estimators of $\bar{Y}$ and $\bar{X}$ at the second phase are respectively given by

$$\bar{y}^* = \frac{n_1 \bar{y}_{n_1} + n_2 \bar{y}_{h_2}}{n}$$

And

$$\bar{x}^* = \frac{n_1 \bar{x}_{n_1} + n_2 \bar{x}_{h_2}}{n}$$

where $\bar{y}_{n_1}$ and $\bar{y}_{h_2}$ are the means based on n_1 responding units and h_2 nonresponding units respectively for the study variable. $\bar{x}_{n_1}$ and $\bar{x}_{h_2}$ are respectively the means based on n_1 responding units and h_2 nonresponding units for the auxiliary variable X . The variances of the estimators $\bar{y}^*$ and $\bar{x}^*$ are respectively given as:

$$V(\bar{y}^*) = \left(\frac{1}{n} - \frac{1}{N} \right) S_Y^2 + \frac{(L-1)}{n} W_2 S_{Y_2}^2$$

$$V(\bar{x}^*) = \left(\frac{1}{n} - \frac{1}{N} \right) S_X^2 + \frac{(L-1)}{n} W_2 S_{X_2}^2$$

where S_Y^2 and S_{Y2}^2 are the population mean squares of entire group and nonresponse respectively for the study variable.

Some Existing Estimators

Chaudhary and Kumar (2016) ^[5] proposed some ratio and regression-type estimators of population means $\bar{Y}$ using the information on the auxiliary variables X under the condition that the nonresponse is observed on both study and auxiliary variables and the population mean $\bar{X}$ of the auxiliary variable X is not known. The estimators are respectively given as

$$T_{1ratio}^* = \frac{\bar{y}^*}{\bar{x}^*} \bar{x}'^*$$

$$T_{2reg}^* = \bar{y}^* + b^*(\bar{x}'^* - \bar{x}^*)$$

where $b^* = \frac{S_{XY}}{S_X^2}$ is an estimator of the population regression coefficient of Y on X i.e. $\beta_1 = \frac{S_{XY}}{S_X^2}$, S_{XY}^2 are respectively the estimators of S_{XY} and S_X^2 based on $(n_1 + h_2)$ units. $S_{XY} = \rho_{XY} S_X S_Y$, and ρ_{XY} is the population correlation coefficient between Y and X . The expression for the mean square error (MSE) of the estimators T_{1ratio}^* and T_{2reg}^* up to the first order of approximation are respectively given as

$$MSE(T_{1ratio}^*) = \left(\frac{1}{n'} - \frac{1}{N}\right) S_Y^2 + \left(\frac{1}{n} - \frac{1}{n'}\right) (S_Y^2 + R^2 S_X^2 - 2\rho_{XY} R S_X S_Y) + \frac{(L-1)}{n} W_2 S_{Y2}^2 + W_2 (R^2 S_{X2}^2 - 2\rho_{XY2} R S_{X2} S_{Y2}) \left[\frac{(L-1)}{n} - \frac{(L'-1)}{n'}\right],$$

And

$$MSE(T_{2ratio}^*) = \left[\left(\frac{1}{n'} - \frac{1}{N}\right) + \left(\frac{1}{n} - \frac{1}{n'}\right)(1 - \rho_{XY}^2)\right] S_Y^2 + \frac{(L-1)}{n} W_2 S_{Y2}^2 + W_2 (\beta_1^2 S_{X2}^2 + 2\beta_1 \rho_{XY2} S_{X2} S_{Y2}) \left[\frac{(L-1)}{n} - \frac{(L'-1)}{n'}\right]$$

where ρ_{XY2} is the population correlation coefficient between Y and X for the nonresponse group. $R = \bar{Y}/\bar{X}$.

Aniying and Enang (2020) ^[2] proposed a product-ratio estimator under double sampling in the presence of nonresponse as follows

$$T_{ae} = \bar{y}_{st} \left(\frac{\bar{x}_{st} + \Phi}{\bar{x}_{st} - \Phi} \right)$$

where $\Phi = \sum_{i=1}^k C_{xi}$, $\bar{y}_{st} = \sum_{i=1}^k p_i \bar{y}_i^*$;

$$\bar{y}_i^* = \frac{n_{i1} \bar{y}_{ni1} + n_{i2} \bar{y}_{ni2}}{n_i}, p_i = \frac{N_i}{N},$$

$\bar{x}_{st} = \sum_{i=1}^k p_i \bar{x}_i$, $\bar{x}_{st}^1 = \sum_{i=1}^k p_i \bar{x}_i^1$, with mean square error given as

$$MSE(T_{ae}) = \left[b^2 R^2 \sum_{i=1}^k \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 S_{yi}^2 \right] - 2abR^2 \right] \sum_{i=1}^k \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 S_{xi}^2 \right] + a^2 R^2 \sum_{i=1}^k \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 S_{xi}^2 \right] - 2bR \sum_{i=1}^k \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \rho_i S_{xi} S_{yi} + 2aR \right] + \sum_{i=1}^k \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \rho_i S_{xi} S_{yi} + \sum_{i=1}^k \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 S_{yi}^2 + \frac{(k_i-1)}{n_i} p_i^2 W_{i2} S_{yi}^2 \right] \right]$$

where

$$R = \frac{\bar{Y}}{\bar{X}}, a = \frac{\bar{X}}{\bar{X} + \Phi}, b = \frac{\bar{X}}{\bar{X} - \Phi}.$$

Chaudhary, Kumar and Vishwakarma (2021) ^[6] proposed some improved estimators of population mean using two- phase sampling scheme in the presence of nonresponse as follows:

$$\hat{T}_{1rat} = \bar{y}^* \frac{\bar{x}_R'^*}{\bar{x}^*} \text{ with}$$

$$MSE(\hat{T}_{1rat}) = \bar{Y}^2 \left[f^*(C_Y^2 + C_X^2 - 2\rho_{XY} C_Y C_X) + f'(C_Y^2 + C_Z^2 - 2\rho_{XZ} C_Y C_Z) + \frac{(L-1)}{n} W_2 \frac{S_{Y2}^2}{\bar{Y}^2} + W_2 \left(\frac{S_{X2}^2}{\bar{X}^2} - 2\rho_{XY2} \frac{S_{X2} S_{Y2}}{\bar{X} \bar{Y}} \right) \left[\frac{(L-1)}{n} - \frac{(L'-1)}{n'} \right] \right]$$

where $f^* = \left(\frac{1}{n} - \frac{1}{n'}\right)$, $f = \left(\frac{1}{n} - \frac{1}{N}\right)$, $f' = \left(\frac{1}{n'} - \frac{1}{N}\right)$

$$\hat{T}_{2reg} = \bar{y}^* + b^*(\bar{x}'_{lr} - \bar{x}^*), \text{ with}$$

$$MSE(\hat{T}_{1rat}) = f'S_Y^2 + \beta_1\beta_2f'(\beta_1\beta_2S_Z^2 - 2\rho_{XY}S_Y S_Z) + \beta_1f^*(\beta_1S_X^2 - 2\rho_{XY}S_Y S_X) + \frac{(L-1)}{n}W_2\frac{S_{Y2}^2}{\bar{Y}^2} + W_2(\beta_1^2S_{X2}^2 - 2\beta_1\rho_{XY2}S_{Y2}S_{X2})\left[\frac{(L-1)}{n} - \frac{(L'-1)}{n'}\right]$$

Anieting *et al.* (2021) [3] proposed an estimator of population mean in stratified double sampling in the presence of non response using single auxiliary variable as follows

$$T_{ae1} = \bar{y}_{st}^* \left(\frac{\bar{x}_{st}^1 - \rho_i}{\bar{x}_{st} - \rho_i} \right)$$

where $\bar{x}_{st} = \sum_{i=1}^k p_i \bar{x}_i$, $\bar{x}_{st}^1 = \sum_{i=1}^k p_i \bar{x}_i^1$, with mean square error given as:

$$MSE(T_{ae1}) = \left[\left(\frac{\bar{x}}{\bar{x} - \rho_i} \right)^2 R^2 \sum_{i=1}^K \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) \rho_i^2 S_{xi}^2 \right] - \left(\frac{\bar{x}}{\bar{x} + \rho_i} \right)^2 R^2 \sum_{i=1}^K \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) P_i^2 S_{xi}^2 \right] - 2R \left(\frac{\bar{x}}{\bar{x} + \rho_i} \right) \sum_{i=1}^K \left(\frac{1}{n_i} - \frac{1}{N_i} \right) P_i^2 \rho_i \rho_i S_{xi} S_{yi} + 2 \left(\frac{\bar{x}}{\bar{x} + \rho_i} \right) R \sum_{i=1}^K \left(\frac{1}{n_i} - \frac{1}{N_i} \right) P_i^2 \rho_i S_{xi} S_{yi} + \sum_{i=1}^K \left[\left(\frac{1}{n_i} - \frac{1}{N_i} \right) \frac{(k_i-1)}{n_i} P_i^2 W_{i2} S_{yi2}^2 \right] \right]$$

Proposed estimators

In this study, some classical ratio and regression type estimators of the population mean $\bar{Y}$ using information on two auxiliary variables are proposed. Under the condition that the study variable Y and auxiliary variable X are affected by nonresponse while the supplementary information on Z is free from the nonresponse. It is further assumed that the population mean $\bar{X}$ is not known. The proposed ratio and regression type estimators under the given situation are respectively represented as follows

$$\hat{T}_{Mra} = \frac{\bar{y}^*}{\bar{x}_R'^*} \bar{x}^*$$

$$\hat{T}_{Mreg} = \bar{y}^* + b^*(\bar{x}^* - \bar{x}'_{lr})$$

where $\bar{x}_R'^* = \frac{\bar{x}'^*}{\bar{Z}}$, $\bar{x}_{lr}'^* = \bar{x}'^* + b'^*(\bar{Z}' - \bar{Z})$ and $b'^* = \frac{S_{XZ}'}{S_Z'^*}$ is an estimator of the population regression coefficient of X and Z i.e $\beta_2 = \frac{S_{XZ}}{S_Z^2}$, S_{XZ}^* and $S_Z'^*$ are respectively the unbiased estimators of S_{XZ} and S_Z^2 , based on $(n'_1 + h'_2)$ units. $S_{XZ} = \rho_{XZ}S_X S_Z$ and ρ_{XZ} is the population correlation coefficient between X and Z .

To obtain the bias and MSEs of $\hat{T}_{1R}$ and $\hat{T}_{2reg}$, we adopt the theory of large sample approximations. Let

$$\bar{y}^* = \bar{Y}(1 + e_0), \bar{x}^* = \bar{X}(1 + e_1), \bar{x}'^* = \bar{X}(1 + e'_1), \bar{Z}' = \bar{Z}(1 + e'_2), s_{xy}^* = S_{XZ}(1 + e_2), s_x^{2*} = S_X^2(1 + e_3), s_{xz}^{t*} = S_{XZ}(1 + e_4), s_Z^{2t*} = S_Z^2(1 + e_5)$$

We first consider Eq.1

$$\begin{aligned} \hat{T}_{Mra} &= \frac{\bar{y}^*}{\bar{x}_R'^*} \bar{x}^* \\ &= \bar{Y}(1 + e_0)(1 + e_1)(1 + e'_1)^{-1}(1 + e'_2)^{-1} \\ &= \bar{Y}[(1 + e_1 + e_0 + e_0e_1)(1 - e'_1 + e_1'^2 - \dots)(1 - e'_2 + e_2'^2 - \dots)] \\ &= \bar{Y}[(1 - e_1^2 - e_1' + e_1'e_2' + e_1'^2 + e_1 - e_1'e_1 + e_0 - e_0e_1' + e_0e_1)] \end{aligned}$$

Taking expectation of the error terms, we have

$$E(e_0) = E(e_1) = E(e_2) = E(e_3) = E(e_4) = E(e_5) = E(e'_1) = E(e'_2) = 0$$

$$E(e_0^2) = \left(\frac{1}{n} - \frac{1}{N} \right) C_Y^2 + \frac{(L-1)}{n} W_2 \frac{S_{Y2}^2}{\bar{Y}^2}$$

$$E(e_1^2) = \left(\frac{1}{n} - \frac{1}{N} \right) C_X^2 + \frac{(L-1)}{n} W_2 \frac{S_{X2}^2}{\bar{X}^2}$$

$$E(e_1'^2) = \left(\frac{1}{n'} - \frac{1}{N} \right) C_X^2 + \frac{(L'-1)}{n'} W_2 \frac{S_{X2}^2}{\bar{X}^2}$$

$$E(e_2'^2) = \left(\frac{1}{n'} - \frac{1}{N}\right) C_Z^2$$

$$E(e_0 e_1) = \left(\frac{1}{n} - \frac{1}{N}\right) \rho_{XY} C_X C_Y + \frac{(L-1)}{n} W_2 \rho_{XY2} \frac{S_{X2}}{\bar{X}} \frac{S_{Y2}}{\bar{Y}}$$

$$E(e_0 e_1') = \left(\frac{1}{n'} - \frac{1}{N}\right) \rho_{XY} C_X C_Y + \frac{(L'-1)}{n'} W_2 \rho_{XY2} \frac{S_{X2}}{\bar{X}} \frac{S_{Y2}}{\bar{Y}}$$

$$E(e_0 e_2') = \left(\frac{1}{n'} - \frac{1}{N}\right) \rho_{YZ} C_Y C_Z$$

$$E(e_1 e_1') = \left(\frac{1}{n'} - \frac{1}{N}\right) C_X^2 + \frac{(L'-1)}{n'} W_2 \frac{S_{X2}^2}{\bar{X}^2}$$

$$E(e_1 e_2') = \left(\frac{1}{n'} - \frac{1}{N}\right) \rho_{XY} C_X C_Y$$

$$E(e_1' e_2') = \left(\frac{1}{n'} - \frac{1}{N}\right) \rho_{XZ} C_X C_Z$$

$$E(e_1 e_2) = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{21}}{n \bar{X} S_{XY}} + W_2 \frac{(L-1)}{n} \frac{\mu_{21(2)}}{\bar{X} S_{XY}}$$

$$E(e_1' e_2) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{21}}{n' \bar{X} S_{XY}} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{21(2)}}{\bar{X} S_{XY}}$$

$$E(e_1 e_3) = \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{30}}{n \bar{X} S_{X2}^2} + W_2 \frac{(L-1)}{n} \frac{\mu_{30(2)}}{\bar{X} S_{X2}^2}$$

$$E(e_1' e_3) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{30}}{n' \bar{X} S_{X2}^2} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{30(2)}}{\bar{X} S_{X2}^2}$$

$$E(e_2' e_2) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{111}}{n' \bar{Z} S_{XY}}$$

$$E(e_2' e_3) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{21}'}{n' \bar{Z} S_X^2}$$

$$E(e_2' e_4) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{12}'}{n' \bar{Z} S_{XY}}$$

$$E(e_2' e_5) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{03}'}{n' \bar{Z} S_Z}$$

$$E(e_1' e_4) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{31}}{n' \bar{X} S_{XZ}} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{31(2)}}{\bar{X} S_{XZ}}$$

$$E(e_1' e_5) = \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{41}}{n' \bar{X} S_Z^2} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{41(2)}}{\bar{X} S_Z^2}$$

where

$$\mu_{rs} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{X})^r (y_i - \bar{Y})^s, \mu_{rs(2)} = \frac{1}{N_2} \sum_{i=1}^{N_2} (x_i - \bar{X}_2)^r (y_i - \bar{Y}_2)^s$$

$$\mu_{rs'} = \frac{1}{N} \sum_{i=1}^N (x_i' - \bar{X})^r (y_i - \bar{Y})^s, \mu_{rst} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{X})^r (y_i - \bar{Y})^s (z_i - \bar{Z})^t,$$

$$C_Y = \frac{S_Y}{\bar{Y}}, C_X = \frac{S_X}{\bar{X}}, C_Z = \frac{S_Z}{\bar{Z}},$$

$$\bar{X}_2 = \frac{1}{N_2} \sum_{i=1}^{N_2} x_i, \bar{Y}_2 = \frac{1}{N_2} \sum_{i=1}^{N_2} y_i$$

and ρ_{XY} is the population correlation coefficient between X and Y . Now Eq.3 can be expressed as

$$\hat{T}_{Mra} - \bar{Y} = \bar{Y} [1 - e_1^2 - e_1' + e_1' e_2' + e_1'^2 + e_1 - e_1' e_1 + e_0 - e_0 e_1' + e_0 e_1 - 1]$$

$$\begin{aligned}
E(\hat{T}_{Mra} - \bar{Y}) &= \bar{Y}[E(e_0) + E(e_1) - E(e'_1) + E(e'_1{}^2) - E(e_1^2) + E(e'_1e'_2) - E(e'_1e_1) - E(e_0e'_1) + E(e_0e_1)] \\
&= \bar{Y}[E(e'_1{}^2) - E(e_1^2) + E(e'_1e'_2) - E(e'_1e_1) - E(e_0e'_1) + E(e_0e_1)] \\
&= \bar{Y}\left[f'C_X^2 + \frac{(L'-1)}{n'}W_2C_{X2}^2 - fC_X^2 - \frac{(L-1)}{n}W_2C_{X2}^2 + f'\rho_{XY}C_XC_Y - f'C_X^2 - \frac{(L'-1)}{n'}W_2C_{X2}^2 - f'\rho_{XY}C_XC_Y - \right. \\
&\quad \left. \frac{(L'-1)}{n'}W_2\rho_{XY}C_XC_Y - f'C_X^2 - \frac{(L'-1)}{n'}W_2C_{X2}^2f'\rho_{XZ}C_XC_Z\right]
\end{aligned}$$

Squaring both sides of Eq.4 and taking expectations

$$\begin{aligned}
E[\hat{T}_{Mra} - \bar{Y}]^2 &= E\{\bar{Y}[e_0 + e_1 - e_1^2 - e'_1 + e_1'^2 + e_0e_1 - e_0e'_1 - e'_1e_1 + e'_1e_2']\}^2 \\
&= \bar{Y}^2 E\{e_0^2 + e_1^2 + e_1'^2 + 2e_0e_1 - 2e_0e'_1 - 2e'_1e_1\} \\
&= \bar{Y}^2 \left\{ \left(\frac{1}{n} - \frac{1}{N}\right)C_Y^2 + \frac{(L-1)}{n}W_2\frac{S_{Y2}^2}{\bar{Y}^2} + \left(\frac{1}{n} - \frac{1}{N}\right)C_X^2 + \frac{(L-1)}{n}W_2\frac{S_{X2}^2}{\bar{X}^2} + \left(\frac{1}{n'} - \frac{1}{N}\right)C_X^2 + \frac{(L'-1)}{n'}W_2\frac{S_{X2}^2}{\bar{X}^2} + 2\left(\frac{1}{n} - \frac{1}{N}\right)\rho_{XY}C_XC_Y + \right. \\
&\quad \left. \frac{(L-1)}{n}W_2\rho_{XY2}\frac{S_{X2}S_{Y2}}{\bar{X}\bar{Y}} - 2\left(\frac{1}{n'} - \frac{1}{N}\right)\rho_{XY}C_XC_Y - \frac{(L'-1)}{n'}W_2\rho_{XY2}\frac{S_{X2}S_{Y2}}{\bar{X}\bar{Y}} - 2\left(\frac{1}{n'} - \frac{1}{N}\right)C_X^2 - \frac{(L'-1)}{n'}W_2\frac{S_{X2}^2}{\bar{X}^2} \right\} \\
&= \bar{Y}^2 \left\{ fC_Y^2 + \frac{(L-1)}{n}W_2C_{Y2}^2 + fC_X^2 + \frac{(L-1)}{n}W_2C_{X2}^2 + f'C_X^2 + \frac{(L'-1)}{n'}W_2C_{X2}^2 + 2f\rho_{XY}C_XC_Y + 2\frac{(L-1)}{n}W_2\rho_{XY2}C_XC_Y - \right. \\
&\quad \left. 2f'\rho_{XY}C_XC_Y - 2\frac{(L'-1)}{n'}W_2\rho_{XY2}C_{X2}C_{Y2} - 2f'C_X^2 - 2\frac{(L'-1)}{n'}W_2C_{X2}^2 \right\} \\
&= \bar{Y}^2 \left\{ fC_Y^2 + C_X^2(f - f') + \frac{(L-1)}{n}W_2(C_{Y2}^2 - C_{X2}^2) - \frac{(L'-1)}{n'}W_2C_{X2}^2 + 2f\rho_{XY}C_XC_Y(f - f') - 2W_2\rho_{XY2}C_XC_Y \left[\frac{(L-1)}{n} - \right. \right. \\
&\quad \left. \left. \frac{(L'-1)}{n'} \right] \right\} \\
&= \bar{Y}^2 \left\{ fC_Y^2 + C_X^2f^* + \frac{(L-1)}{n}W_2C_{Y2}^2 + 2f\rho_{XY2}C_XC_Yf^* - W_2C_{X2}^2 \left[\frac{(L-1)}{n} - \frac{(L'-1)}{n'} \right] - 2W_2\rho_{XY2}C_{X2}C_{Y2} \left[\frac{(L-1)}{n} - \frac{(L'-1)}{n'} \right] \right\} \\
MSE(\hat{T}_{Mra}) &= \bar{Y}^2
\end{aligned}$$

Where

$$f^* = \left(\frac{1}{n} - \frac{1}{n^1}\right) \text{ and } f_2 = \left[\frac{(L-1)}{n} - \frac{(L'-1)}{n'}\right]$$

Consider Eq.2

$$\begin{aligned}
\hat{T}_{Mreg} &= \bar{y}^* + b^*(\bar{x}^* - \bar{x}'_{br}) \\
&= \bar{y}^* + b^*[\bar{x}^* - \{\bar{x}'^* - b^{1*}(\bar{z}' - \bar{Z})\}] \\
&= \bar{Y}(1 + e_0) + \frac{S_{XY}(1+e_2)}{S_X^2(1+e_3)} \left[\bar{X}(1 + e_1) - \left\{ \bar{X}(1 + e'_1) - \frac{S_{XZ}(1+e_4)}{S_Z^2(1+e_5)}(\bar{Z}[1 + e'_2] - \bar{Z}) \right\} \right] \\
&= \bar{Y} + \bar{Y}e_0 + \beta_1(1 + e_2)(1 + e_3)^{-1}[\bar{X}e_1 - \bar{X}e'_1 + \beta_2(1 - e_5 + e_5^2 + e_4 - e_4e_5)(\bar{Z}e_2)] \\
&= \bar{Y} + \bar{Y}e_0 + \beta_1\bar{X}[e_1 - e_1e_3 + e_1e_2 - e'_1 + e'_1e_3 - e'_1e_2] + \beta_1\beta_2\bar{Z}[(e'_1 - e'_1e_5 + e'_1e_4 - e'_1e_3 + e'_1e_2)] \\
\hat{T}_{Mreg} - \bar{Y} &= \bar{Y}e_0 + \beta_1\bar{X}[e_1 - e_1e_3 + e_1e_2 - e'_1 + e'_1e_3 - e'_1e_2] + \beta_1\beta_2\bar{Z}[(e'_1 - e'_1e_5 + e'_1e_4 - e'_1e_3 + e'_1e_2)] \\
E(\hat{T}_{Mreg} - \bar{Y}) &= \bar{Y}E(e_0) + \beta_1\bar{X}[E(e_1) - E(e_1e_3) + E(e_1e_2) - E(e'_1) + E(e'_1e_3) - E(e'_1e_2)] + \beta_1\beta_2\bar{Z}[(E(e'_1) - E(e'_1e_5) + E(e'_1e_4) - E(e'_1e_3) + E(e'_1e_2))] \\
&= \beta_1\bar{X}[-E(e_1e_3) + E(e_1e_2) + E(e'_1e_3) - E(e'_1e_2)] + \beta_1\beta_2\bar{Z}[(-E(e'_1e_5) + E(e'_1e_4) - E(e'_1e_3) + E(e'_1e_2))] \\
&= \beta_1\bar{X} \left[-\frac{N(N-n)}{(N-1)(N-2)}\frac{\mu_{30}}{n\bar{X}S_X^2} - W_2\frac{(L-1)}{n}\frac{\mu_{30(2)}}{\bar{X}S_X^2} + \frac{N(N-n)}{(N-1)(N-2)}\frac{\mu_{21}}{n\bar{X}S_{XY}} + W_2\frac{(L-1)}{n}\frac{\mu_{21(2)}}{\bar{X}S_{XY}} + \frac{N(N-n')}{(N-1)(N-2)}\frac{\mu_{30}}{n'\bar{X}S_X^2} + W_2\frac{(L'-1)}{n'}\frac{\mu_{30(2)}}{\bar{X}S_X^2} - \right. \\
&\quad \left. \frac{N(N-n')}{(N-1)(N-2)}\frac{\mu_{21}}{n'\bar{X}S_{XY}} - W_2\frac{(L'-1)}{n'}\frac{\mu_{21(2)}}{\bar{X}S_{XY}} \right] + \beta_1\beta_2\bar{Z} \left[\left(-\frac{N(N-n')}{(N-1)(N-2)}\frac{\mu_{41}}{n'\bar{Z}S_Z^2} - W_2\frac{(L'-1)}{n'}\frac{\mu_{41(2)}}{\bar{Z}S_Z^2} + \frac{N(N-n')}{(N-1)(N-2)}\frac{\mu_{31}}{n'\bar{Z}S_{XZ}} + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& W_2 \frac{(L'-1)}{n'} \frac{\mu_{31(2)}}{\bar{Z}S_{XZ}} - \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{30}}{n' \bar{X}S_X^2} - W_2 \frac{(L'-1)}{n'} \frac{\mu_{30(2)}}{\bar{X}S_X^2} + \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{21}}{n' \bar{X}S_{XY}} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{21(2)}}{\bar{X}S_{XY}} \Big] \\
& = \beta_1 \left[-\frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{30}}{n S_X^2} - W_2 \frac{(L-1)}{n} \frac{\mu_{30(2)}}{S_X^2} + \frac{N(N-n)}{(N-1)(N-2)} \frac{\mu_{21}}{n S_{XY}} + W_2 \frac{(L-1)}{n} \frac{\mu_{21(2)}}{S_{XY}} + \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{30}}{n' S_X^2} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{30(2)}}{S_X^2} - \right. \\
& \quad \left. \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{21}}{n' S_{XY}} - W_2 \frac{(L'-1)}{n'} \frac{\mu_{21(2)}}{S_{XY}} \right] + \beta_1 \beta_2 \left[\left(-\frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{41}}{n' S_Z^2} - W_2 \frac{(L'-1)}{n'} \frac{\mu_{41(2)}}{S_Z^2} + \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{31}}{n' S_{XZ}} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{31(2)}}{S_{XZ}} - \right. \right. \\
& \quad \left. \left. \frac{\bar{Z}N(N-n')}{(N-1)(N-2)} \frac{\mu_{30}}{n' \bar{X}S_X^2} - W_2 \frac{(L'-1)\bar{Z}}{n'} \frac{\mu_{30(2)}}{\bar{X}S_X^2} + \frac{\bar{Z}N(N-n')}{(N-1)(N-2)} \frac{\mu_{21}}{n' \bar{X}S_{XY}} + W_2 \frac{(L'-1)\bar{Z}}{n'} \frac{\mu_{21(2)}}{\bar{X}S_{XY}} \right) \right] \\
& = \beta_1 \left[\left(\frac{N^2}{(N-1)(N-2)} \right) \left(\frac{n'-n}{nn'} \right) \left(\frac{\mu_{21}}{S_{XY}} \right) - \left(\frac{N^2}{(N-1)(N-2)} \right) \left(\frac{n'-n}{nn'} \right) \left(\frac{\mu_{30}}{S_X^2} \right) \right] + \beta_1 \beta_2 \left[\left(-\frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{41}}{n' S_Z^2} - W_2 \frac{(L'-1)}{n'} \frac{\mu_{41(2)}}{S_Z^2} + \right. \right. \\
& \quad \left. \left. \frac{N(N-n')}{(N-1)(N-2)} \frac{\mu_{31}}{n' S_{XZ}} + W_2 \frac{(L'-1)}{n'} \frac{\mu_{31(2)}}{S_{XZ}} - \frac{\bar{Z}N(N-n')}{(N-1)(N-2)} \frac{\mu_{30}}{n' \bar{X}S_X^2} - W_2 \frac{(L'-1)\bar{Z}}{n'} \frac{\mu_{30(2)}}{\bar{X}S_X^2} + \frac{\bar{Z}N(N-n')}{(N-1)(N-2)} \frac{\mu_{21}}{n' \bar{X}S_{XY}} + W_2 \frac{(L'-1)\bar{Z}}{n'} \frac{\mu_{21(2)}}{\bar{X}S_{XY}} \right) \right] \\
& \text{Bias}(\hat{T}_{Mreg}) = \beta_1 \left[\left(\frac{N^2}{(N-1)(N-2)} \right) \left(\frac{n'-n}{nn'} \right) \left[\frac{\mu_{21}}{S_{XY}} - \frac{\mu_{30}}{S_X^2} \right] + W_2 \left(\frac{(L'-1)}{n'} - \frac{(L-1)}{n} \right) \left(\frac{\mu_{30(2)}}{S_X^2} - \frac{\mu_{21(2)}}{S_{XY}} \right) \right] + \\
& \beta_1 \beta_2 \left[\left(-\frac{N(N-n')}{(N-1)(N-2)} \left[\frac{\mu_{41}}{S_Z^2} - \frac{\mu_{31(2)}}{S_{XZ}} \right] + W_2 \frac{(L'-1)}{n'} \left[\frac{\mu_{31}}{S_{XZ}} - \frac{\mu_{41(2)}}{S_Z^2} \right] - R^* \left\{ \frac{N(N-n')}{(N-1)(N-2)} \left[\frac{\mu_{21}}{S_{XY}} - \frac{\mu_{30}}{S_X^2} \right] + W_2 \frac{(L'-1)}{n'} \left[\frac{\mu_{21(2)}}{S_{XY}} - \right. \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\mu_{30(2)}}{S_X^2} \right] \right\} \right] \Big]
\end{aligned}$$

where $R^* = \frac{\bar{Z}}{\bar{X}}$

To obtained the mean square error, we have

$$\begin{aligned}
E(\hat{T}_{Mreg} - \bar{Y})^2 &= E[\bar{Y}e_0 + \beta_1 \bar{X}[e_1 - e_1 e_3 + e_1 e_2 - e_1' + e_1' e_3 - e_1' e_2] + \beta_1 \beta_2 \bar{Z}[(e_1' - e_1' e_5 + e_1' e_4 - e_1' e_3 + e_1' e_2)]]^2 \\
&= E[\bar{Y}^2 e_0^2 + \beta_1 \bar{Y} \bar{X}[e_0 e_1 - e_0 e_1'] + \beta_1 \beta_2 \bar{Y} \bar{Z}(e_0 e_1')] + \beta_1 \bar{Y} \bar{X}[e_0 e_1 - e_0 e_1'] + \beta_1^2 \bar{X}^2(e_1' - e_1 e_1' - e_1 e_1' + e_1'^2) + \\
& \quad \beta_1^2 \beta_2 \bar{Y} \bar{Z}(e_1 e_1' - e_1'^2) + \beta_1 \beta_2 \bar{Y} \bar{Z}(e_0 e_1') + \beta_1^2 \beta_2 \bar{Y} \bar{Z}(e_1 e_1' - e_1'^2) + \beta_1^2 \beta_2^2 \bar{Z}^2 e_1'^2] \\
&= [\bar{Y}^2 E(e_0^2) + 2\beta_1 \bar{Y} \bar{X}[E(e_0 e_1) - E(e_0 e_1')] + 2\beta_1 \beta_2 \bar{Y} \bar{X} E(e_0 e_1')] + \beta_1^2 \bar{X}^2[E(e_1') - 2E(e_1 e_1') + E(e_1'^2)] + \\
& \quad 2\beta_1^2 \beta_2 \bar{Y} \bar{Z}[E(e_1 e_1') - E(e_1'^2)] + \beta_1^2 \beta_2^2 \bar{Z}^2 E(e_1'^2)] \\
&= \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{N} \right) C_Y^2 + \frac{(L-1)}{n} W_2 \frac{S_{Y2}^2}{\bar{Y}^2} + 2\beta_1 \bar{Y} \bar{X} \left(\frac{1}{n'} - \frac{1}{N} \right) \rho_{XY} C_X C_Y + \frac{(L-1)}{n} W_2 \rho_{XY2} \frac{S_{X2}}{\bar{X}} \frac{S_{Y2}}{\bar{Y}} - 2\beta_1 \bar{Y} \bar{X} \left(\frac{1}{n'} - \frac{1}{N} \right) \rho_{XY} C_X C_Y - \\
& \quad \frac{(L'-1)}{n'} W_2 \rho_{XY2} \frac{S_{X2}}{\bar{X}} \frac{S_{Y2}}{\bar{Y}} + 2\beta_1 \beta_2 \bar{Y} \bar{Z} \left(\frac{1}{n'} - \frac{1}{N} \right) \rho_{XY} C_X C_Y + \frac{(L'-1)}{n'} W_2 \rho_{XY2} \frac{S_{X2}}{\bar{X}} \frac{S_{Y2}}{\bar{Y}} + \beta_1^2 \bar{X}^2 \left(\frac{1}{n} - \frac{1}{N} \right) C_X^2 + \frac{(L-1)}{n} W_2 \frac{S_{X2}^2}{\bar{X}^2} - \\
& \quad \beta_1^2 \bar{X}^2 \left(\frac{1}{n'} - \frac{1}{N} \right) C_X^2 - \frac{(L'-1)}{n'} W_2 \frac{S_{X2}^2}{\bar{X}^2} + \beta_1^2 \bar{X}^2 \left(\frac{1}{n'} - \frac{1}{N} \right) C_X^2 + \frac{(L'-1)}{n'} W_2 \frac{S_{X2}^2}{\bar{X}^2} + 2\beta_1^2 \beta_2 \bar{Y} \bar{Z} \left(\frac{1}{n'} - \frac{1}{N} \right) C_X^2 + \frac{(L'-1)}{n'} W_2 \frac{S_{X2}^2}{\bar{X}^2} - \left(\frac{1}{n'} - \right. \\
& \quad \left. \frac{1}{N} \right) C_X^2 - \frac{(L'-1)}{n'} W_2 \frac{S_{X2}^2}{\bar{X}^2} + \beta_1^2 \beta_2^2 \bar{Z}^2 \left(\frac{1}{n'} - \frac{1}{N} \right) C_X^2 + \frac{(L'-1)}{n'} W_2 \frac{S_{X2}^2}{\bar{X}^2} \\
&= \left(\frac{1}{n} - \frac{1}{N} \right) \{ \bar{Y}^2 C_Y^2 + 2\beta_1 \bar{Y} \bar{X} \rho_{XY} C_X C_Y + \beta_1^2 \bar{X}^2 C_X^2 \} - \frac{(L-1)}{n} W_2 \left\{ \frac{S_{Y2}^2}{\bar{Y}^2} + \rho_{XY2} \frac{S_{X2}}{\bar{X}} \frac{S_{Y2}}{\bar{Y}} + \frac{S_{X2}^2}{\bar{X}^2} \right\} + \frac{(L'-1)}{n'} 2W_2 \frac{S_{X2}^2}{\bar{X}^2} + \left(\frac{1}{n'} - \right. \\
& \quad \left. \frac{1}{N} \right) \{ 2\beta_1 \bar{Y} (\beta_2 \bar{Z} \rho_{XY} C_X C_Y - \bar{X} \rho_{XY} C_X C_Y) \} - \left(\frac{1}{n'} - \frac{1}{N} \right) \{ \beta_2^2 (\bar{Z} C_X^2 - \bar{X}^2 C_X^2) \} \\
& \text{MSE}(\hat{T}_{Mreg}) = f \{ \bar{Y}^2 C_Y^2 + 2\beta_1 \bar{Y} \bar{X} \rho_{XY} C_X C_Y + \beta_1^2 \bar{X}^2 C_X^2 \} - \frac{(L-1)}{n} W_2 \left\{ \frac{S_{Y2}^2}{\bar{Y}^2} + \rho_{XY2} \frac{S_{X2}}{\bar{X}} \frac{S_{Y2}}{\bar{Y}} + \frac{S_{X2}^2}{\bar{X}^2} \right\} + \frac{(L'-1)}{n'} 2W_2 \frac{S_{X2}^2}{\bar{X}^2} + \\
& \quad f^* \{ 2\beta_1 \bar{Y} (\beta_2 \bar{Z} \rho_{XY} C_X C_Y - \bar{X} \rho_{XY} C_X C_Y) \} - f^* \{ \beta_2^2 (\bar{Z} C_X^2 - \bar{X}^2 C_X^2) \}
\end{aligned}$$

Empirical study

In order to examine the behaviour of the proposed estimators. It is essential to investigate the theoretical claims with empirical analysis. Therefore, we have considered two different data sets with a view to establish acceptable results.

Date Set 1

Data used is from population I considered by Anderson (1958) ^[1]. Here, the head length of the second sum is considered as the study variable Y, whereas the head length of the first sum and the head breadth of the first sum are respectively considered as the auxiliary X and Z. The population details are given below:

$$N = 25, n' = 10, n = 7, \bar{X} = 185.72, \bar{Y} = 183.84, \bar{Z} = 151.121, C_X = 0.0526, C_Y = 0.0546,$$

$$C_Z = 0.0488, S_X = 9.78872, S_Y = 10.03766, S_Z = 7.374656, S_{X2} = 6.512581,$$

$$S_{Y2} = 6.691776, S_{Z2} = 4.91647, \rho_{XY} = 0.7108, \rho_{XZ} = 0.6932, \rho_{YZ} = 0.7346,$$

$$\rho_{XY2} = 0.473867.$$

Table 1: Variance/MSE of the Estimators

W_2	$L = L'$	Variance / MSE								
0.2		$\bar{y}^*$	T_{1rat}^*	T_{1rat}^{**}	T_{2reg}^*	T_{2reg}^{**}	T_{ae}	T_{ae1}	$\hat{T}_{Mra}$	$\hat{T}_{Mreg}$
	1.5	11.0	9.04	6.4	8.7	6.0	5.3	5.0	4.51	4.00
	2	11.6	9.68	7.1	9.4	6.6	5.9	5.1	4.58	4.02
	2.5	12.2	10.3	7.7	10.0	7.2	6.1	5.2	5.33	5.13
	3	12.9	10.9	8.3	10.6	7.8	6.7	5.3	5.87	5.32

Table 2: PRE of the Estimators

W_2	$L = L'$	PRE								
0.2		$\bar{y}^*$	T_{1rat}^*	T_{1rat}^{**}	T_{2reg}^*	T_{2reg}^{**}	T_{ae}	T_{ae1}	$\hat{T}_{Mra}$	$\hat{T}_{Mreg}$
	1.5	100	121.8	170.2	125.2	181.5	190.5	193.3	195.1	211.6
	2	100	120.3	164.4	123.9	174.6	189.3	190.6	192.5	203.4
	2.5	100	118.0	159.0	122.8	168.9	182.7	187.4	188.9	200.9
	3	100	117.8	154.5	121.8	163.9	173.3	184.8	185.7	198.3

Data Set 2

Here, the data considered by Singh (1967) is used to demonstrate the theoretical results. In this data set, number of females is considered to be the study variable Y , whereas the number of females in service and number of educated females are considered to be auxiliary variable X and Z respectively. The characteristics of the population are given below:

$$N = 61, n' = 20, n = 10, \bar{X} = 5.31, \bar{Y} = 7.46, \bar{Z} = 179.00, C_X = 0.7587, C_Y = 0.71303,$$

$$C_Z = 0.2515, S_X = 4.028697, S_Y = 5.298838, S_Z = 45.0185, S_{X2} = 2.685798, S_{Y2} = 3.532559,$$

$$S_{Z2} = 30.01233, \rho_{XY} = 0.7737, \rho_{XZ} = 0.207, \rho_{YZ} = 0.0033, \rho_{XY2} = 0.5158.$$

Table 3: Variance/MSE of the Estimators

W_2	$L = L'$	Variance / MSE								
0.2		$\bar{y}^*$	T_{1rat}^*	T_{1rat}^{**}	T_{2reg}^*	T_{2reg}^{**}	T_{ae}	T_{ae1}	$\hat{T}_{Mra}$	$\hat{T}_{Mreg}$
	1.5	2.4	2.4	2.0	1.6	1.6	2.5	2.3	1.9	1.2
	2	2.5	2.6	2.1	1.7	1.7	2.6	2.4	2.0	1.2
	2.5	2.7	2.6	2.2	1.8	1.8	2.7	2.6	2.0	1.3
	3	2.8	2.8	2.4	1.9	1.9	2.8	2.7	2.1	1.4

Table 4: PRE of the Estimators

W_2	$L = L'$	PRE								
0.2		$\bar{y}^*$	T_{1rat}^*	T_{1rat}^{**}	T_{2reg}^*	T_{2reg}^{**}	T_{ae}	T_{ae1}	$\hat{T}_{Mra}$	$\hat{T}_{Mreg}$
	1.5	100	102	123	153	100	121	153	135	165
	2	100	102	121	150	100	199	150	133	163
	2.5	100	102	120	148	100	199	148	132	163
	3	100	101	119	146	99	198	146	133	161

Conclusion

Two estimators have been proposed in the form of the classical ratio and regression-type estimators of the population mean using the information on two auxiliary variables under double sampling scheme with subsampling the nonrespondents. A theoretical study on the properties of the proposed estimators has been presented in details. A comparative study of the proposed estimators with usual mean estimators and some existing estimators of the population mean have also been presented. To support the theoretical results, empirical study has been considered with two different real data sets. From the Tables 1a, 1b, 2a and 2b, it is observed that the proposed estimators $\hat{T}_{Mra}$ and $\hat{T}_{Mreg}$ performed better than the existing estimators. This study has proffer solution to the inherent challenges encountered during estimation of survey data such as; health statistics, household income and expenditure, etc, which involve sensitive indicators that might attract nonresponse. This study has established the necessity for researchers and users of statistics to adopt the proposed estimators for precise and efficient estimates.

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