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On the Geometry of Kenmotsu Manifolds with Semi-Symmetric Non-Metric Connection: A Comprehensive Review

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Abstract

Our goal in this review study is to provide a thorough and mostly self-contained description of semi-symmetric non-metric connections on Kenmotsu manifolds. Kenmotsu manifolds are a special class of nearly contact metric manifold, also an interesting class of nearly contact structures that has relevance in the differential geometry itself due to its rich geometrical nature and several theoretical and practical applications. This means that classical Riemannian geometry can be generalized in such a way that we make the condition of metric compatibility less strict, while still having some aspects of torsion retained.

The main objective of this review is to explain some basic concepts from all these angles based on fundamental definitions and features coming from curvature tensors as well the Ricci and scalar behaviors, invariance conditions; with strong innovations in the field. We focus on the case of some Gauduchon-type metrics as Ricci solitons, η -Einstein manifolds and different curvature tensors. In addition, the provided survey emphasizes important results and open challenges along with promising directions for future studies.

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1. Introduction

In recent decades, a great deal of work has been done studying contact geometry and associated structures. Within this framework, the connections play a critical role in symbolizing the geometric properties of manifolds. In many geometrical situations, one has to deal with wider classes of affine connections, even though the Levi-Civita relationship is metric-compatible and torsion-free. One such generalization, is provided by the semi-symmetric non-metric connections that allow embodiment of torsion and non-metricity on geometrical structures. Such connections, first explored by Hayden in 1932 and further developed through Yano, provide a rich source of geometric analysis and more powerful techniques for understanding more subtle aspects of the structures on manifolds.

In this regard, the class of Kenmotsu manifolds, first described by Kenmotsu in 1972, is a particular subclass of nearly contact metric manifolds with a requirement on the covariant derivative of the structure tensor. Conversely, Friedmann and Schouten (1924) proposed semi-symmetric connections, which Agashe & Chafle subsequently expanded to semi-symmetric non-metric connections. The connections loosen the demand for metric compatibility, yielding a more general geometric structure that can account for more general types of curvature.

One of the richest scenarios for geometry is presented by Kenmotsu structures interacting with semi-symmetric non-metric connections. We aim to review, gather and systematize results combining them into a common accessible framework which we believe is both necessary for research, and valuable at the graduate student level.

2. Preliminary preparatory measure

2.1. Almost Contact Metric Manifolds

A virtually contact metric manifold is a $(2n+1)$ -dimensional smooth manifold M with a structure (ϕ, ξ, η, g) that meets the following requirements:

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1,$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y).$$

In this case, g is a Riemannian metric, η is a vector field known as the Reeb vector field, η is a 1-form, and η is a $(1,1)$ tensor field.

2.2. Kenmotsu Manifolds

A Kenmotsu manifold is characterized by the condition:

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$$

The Levi-Civita link is shown by ∇ for any vector fields X, Y on M .

Equivalent conditions include:

$$\nabla_X \xi = X - \eta(X)\xi.$$

2.3 Semi-Symmetric Non-Metric Connection

If the torsion tensor T of a linear connection $\tilde{\nabla}$ on a manifold meets the following criteria, it is considered semi-symmetric.

$$T(X, Y) = \eta(Y)X - \eta(X)Y.$$

for η in 1-form.

It is called non-metric if:

$$\tilde{\nabla} g \neq 0.$$

A semi-symmetric non-metric relationship is often expressed as follows:

$$\tilde{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X, Y)\xi.$$

where the Levi-Civita link is denoted by ∇ .

3. Curvature Properties

3.1. Tensor of Riemannian Curvature

The following is the definition of the curvature tensor connected to the link $\tilde{\nabla}$:

$$\tilde{R}(X, Y)Z = \tilde{\nabla}_X \tilde{\nabla}_Y Z - \tilde{\nabla}_Y \tilde{\nabla}_X Z - \tilde{\nabla}_{[X, Y]}Z.$$

Relationships between the Levi-Civita curvature tensor R and the modified curvature tensor $\tilde{R}$ are calculated, taking into account extra torsion-related factors.

3.2 Ricci Tensor and Scalar Curvature

By contraction, one may derive the Ricci tensor $\tilde{S}$:

$$\tilde{S}(X, Y) = \text{trace}\{Z \rightarrow \tilde{R}(Z, X)Y\}.$$

Key relationships involve:

- The alteration of Ricci curvature arising from non-metricity.

- The dependence of scalar curvature on torsion.

4. Kenmotsu Manifolds under Semi-Symmetric Non-Metric Connection

4.1. Basic Identities

The torsion, non-metricity and structural compatibility of a semi-symmetric non-metric relationship on Kenmotsu manifold satisfy the following defining relations:

$$\tilde{\nabla}_X \xi = X - 2\eta(X)\xi$$

which differs from the classical Levi-Civita case.

4.2. Curvature Relations

The curvature tensor satisfies modified (or generalized) identities such as:

$$\tilde{R}(X, Y)\xi = \eta(X)Y - \eta(Y)X.$$

Such relations are important: they expose the torsion influence.

5. Special Geometric Structures

5.1. η -Einstein Manifolds

A η -Einstein Kenmotsu manifold is defined as follows:

$$S = ag + b\eta \otimes \eta.$$

Under semi-symmetric non-metric connections, this condition undergoes a major transformation.

5.2. Conformal and Projective Curvature

These geometric relationships have been studied extensively using conformal curvature tensors, leading to many remarkable structural results. Using results for Conformal resp. Projective Flatness, one obtains important classification outcomes of Kervaire–Milnor type. These results aid in characterizing manifolds that have some notion of curvature rigidity, elucidate the equivalence classes of geometric structures and provide insight into how constraints on curvature affect topology and differential geometric phenomena.

5.3. Ricci Solitons

Ricci solitons satisfy:

$$\mathcal{L}_V g + 2S + 2\lambda g = 0.$$

Indeed, under these structures, Kenmotsu manifolds supporting Ricci solitons exhibit strong rigidity features by recent works.

6. Invariant Conditions and Symmetry

In differential geometry, great interest has been devoted to the class of manifolds which fulfill certain curvature conditions such as locally symmetric manifolds, semi-symmetric spaces and Ricci-recurrent manifolds. In fact, each of the curvature conditions places strong restrictions on the underlying geometry, including the behavior of curvature, symmetries and ultimately a classification of all manifolds. They also lead to the study of special connections and curvature identities therein.

7. Recent Developments

Various models that have been proposed to account for the mechanism, have primarily focused on conformal curvature tensors, projective curvature properties and their respective geometric interpretation and/or potential repercussions within differential geometry or aspect fields of theoretical physics.

8. Open Problems and Future Directions

Some of the directions for future research include:

1. Generalized connections and classification of Kenmotsu manifolds
2. Fraction numeric structure reason run-life into The.
3. Exploring applications to mathematical physics models.

9. Conclusion

In this review, we objective to give a inclusive account of the province-of-the-art results on semi-symmetric non-metric connections in Kenmotsu manifolds. We invoke the geometric structure that arises through an analysis of the coupling between torsion, non-metricity and their associated contact structure. Such insights provide rich curvature behavior, torsion-free scenarios and classification theorems that shed light on the differential geometry of the manifold itself that may be a precursor to future theoretical advancements.

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