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Genetic Algorithms for Optimization Involving Optimal Control and Transfer

Ali Fahem Abbas
Ministry of Education, Babylon Education Directorate, Iraq

* Corresponding Author: **Ali Fahem Abbas**

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Abstract
This research introduces a cohesive framework that amalgamates the ideas of optimum transfer and optimal control inside genetic algorithms, aimed at enhancing methodologies for addressing intricate optimization challenges. The proposed framework utilizes optimum transfer ideas to enhance mating and crossing processes in genetic algorithms, leveraging optimal control theory to direct the search process and modify algorithm parameters over generations. The proposed methodology was implemented in four distinct case studies: production scheduling, transport network design, resource allocation in cloud computing systems, and investment portfolio optimization. The findings demonstrated the superiority of the suggested methodology compared to typical genetic algorithms for solution quality, convergence speed, and the capacity to evade local optima. An exhaustive evaluation of computational performance and efficiency was provided, along with suggestions for the methodology's application in additional domains and its prospective advancement.

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Keywords: Optimum Control, Optimum Transportation, Genetics

1. Introduction

Genetic algorithms are a type of evolutionary research approach that has found widespread use in tackling complex optimization issues across many different industries [1]. These algorithms are motivated by the notion of natural evolution. Using the tenets of natural selection, reproduction, and mutations, these algorithms build a population of potential solutions (individuals) [2].

While genetic algorithms have shown effective in numerous domains, they do encounter certain obstacles, such as sluggish convergence, slipping into locally optimal solutions, and trouble with parameter tuning [3]. However, for dynamic system routing and resource transfer, respectively, the optimal control theory and the optimal transport theory give robust mathematical foundations [4].

In order to better solve complicated optimization problems, this research article will investigate the potential of combining genetic algorithms with the ideas of optimum control and optimal transfer [5].

1.1. genetic algorithms

Genetic algorithms are evolutionary methodologies that simulate the mechanisms of natural selection. The algorithm initiates with a collection of potential solutions (population) and executes a sequence of operations: selection, crossover, and mutation, with the objective of enhancing solution quality via succeeding generations [6, 7].

The conventional genetic algorithm's fundamental steps are:

- Initialization is the process of randomly generating an initial population of potential solutions, or chromosomes.
- **Evaluation:** the determination of each population member's fitness value.
- **Selection:** the process of choosing the best candidates for reproduction.
- Mating is the process by which the genetic traits of the chosen parents are combined to create new individuals, or

offspring.
- Making minor, arbitrary modifications to certain people is called mutation.
- **Replacement:** bringing in new people to replace a portion of the current population.
- Steps 2 through 6 should be repeated until the Stop condition is met.

1.2. optimal transport theory
The optimal transfer theory, developed by the French mathematician Gaspard Monge, seeks to find the cheapest way to move mass from one probability distribution to another. We aim to determine the transport plan γ that does the following in the mathematical formulation, given two probability distributions μ and V [7, 8]:

$$\min_{\gamma\in\Gamma(\mu,V)} \int_{X\times Y} c(x,y)d\gamma(x,y)$$

where,
- Γ (μ, V) is the set of all possible transport plans that converts μ to V.
- $c\ (x,y)$ is a cost function that represents the cost of moving a unit of mass from Point x to point Y.

The Wasserstein distance, a robust mathematical tool for comparing distributions, is provided by optimal transport theory.

1.3. Optimal Control Theory
The goal of optimal control theory is to maximize (or minimize) a target function by identifying the control signals that move a dynamical system from an initial state to a final one. The optimal control problem is generally stated as follows [9, 10]:

$$\min_{u(.)} J(u) = \int_{t_0}^{t_f} L(x(t),u(t),t)dt + \varphi(x(t_f),t_f)$$

Constraints

$$\dot{x}(t) = f(x(t),u(t),t)$$

$$x(t_0) = x_0$$

where,
- $x\ (t)$ is the state of the system.
- $u(t)$ are the control variables.
- L is the instantaneous cost function.
- φ is a function of the final cost.
- f is a function that describes the dynamics of a system.

1.4. integration of optimal transfer and optimal control of genetic algorithms
We suggest a comprehensive framework that incorporates the theories of optimal transmission and optimal control of genetic algorithms. The frame is composed of two primary components [11, 12]:

1. Optimal transfer in mating and crossing processes: employing the principles of optimal transfer to enhance mating processes by conceptualizing chromosomes as probability distributions and endeavoring to transfer the "genetic mass" at minimal cost.
2. Optimal regulation of the research process orientation: employing optimal control theory to direct the research process and modify algorithm parameters (such as mating and mutation rates) across generations, by representing population development as a dynamic system.

1.4.1. Optimal Transportation in Mating and Crossing Processes
In conventional genetic algorithms, the mating (crossover) process occurs in a predominantly stochastic manner, potentially resulting in the loss of critical information or the fragmentation of beneficial genetic configurations [13, 14]. We propose employing optimal transportation principles to enhance the mating process.

We regard each chromosome as a probability distribution throughout the gene space and create a cost function $c(x,y)$ that quantifies the "cost" of relocating a gene from position x in the first chromosome to place y in the second chromosome. This cost can be determined by metrics of distance within the problem space or by metrics of similarity among genes.

Upon defining the cost function, we address the optimal transfer problem to derive a γ transfer plan, which specifies the gene combination from parental chromosomes to generate daughter chromosomes. This method guarantees the maintenance of beneficial genetic configurations and diversity within the population.

1.4.2. Optimal Control Over the Direction of the Search Process
Using a genetic algorithm to model population evolution as a dynamic system, where [15, 16]:
- The state variables $x(t)$ represents the distribution of the population in generation T.
- Control variables $u(t)$ represent the parameters of the algorithm (mating and mutation rates, selection strategies, etc.).
- The dynamics of system f describes how a population evolves from one generation to another.
- The objective function J represents the quality of solutions and the diversity in the population.

We find the best control strategy $u^*(t)$ that makes the algorithm work as well as possible by applying the optimal control theorem. By adjusting the algorithm parameters in real-time according to the search state, this method makes the search more efficient and prevents the algorithm from getting stuck in local optimal solutions.

2. Proposed Methodology
2.1. Optimized Genetic Algorithm with Optimal Transfer and Optimal Control (OTCGA)
An enhanced genetic algorithm is presented, which incorporates the principles of optimal transport alongside the Control-enhanced Genetic Algorithm (OTCGA). The procedure is comprised of the subsequent steps:

1. **Configuration:**
- Create an initial population P_0 of N individuals randomly.
- Initialization of the initial control parameters u_0 (mating

and mutation rates, selection strategies).

2. **Key Ring: for Each Generation t = 0, 1, 2, ...**

- **Evaluation:** calculation of the fitness value $f(x)$ for each individual $x \in P_t$.
- **Research status analysis:** calculation of research status indicators (average fitness, best fitness, diversity in the population).
- **Adjust control parameters:** update u_{t+1} control parameters based on the optimal control model.
- **Selection:** selection of the parent group S from P_t using the selected selection strategy.
- **Optimized transfer-optimized mating:** for each pair of parents $(x, y) \in S$:
  - ➢ Representation of chromosomes as probability distributions.
  - ➢ Definition of the cost function $c(i, j)$ based on measures of distance or similarity.
  - ➢ Solve the optimal transport problem to obtain the transport plan γ.
  - ➢ Production of offspring using the transport plan γ.
- **Mutation:** performing mutations on children at the specified mutation rate.
- **Replacement:** replacement of part of the existing population with new ones.
- **Redundancy:** transition to the next generation $t + 1$.

3. **Stop condition:** stop when the maximum number of generations is reached or when the best solution for a certain number of generations is not optimized.

2.2. Details of Optimal Transportation in the Mating Process

To implement the optimal transfer in the mating process, we adhere to these steps:

- Chromosome representation: we represent the parental X and y chromosomes as the μ and V probability distributions over the gene space.
- Definition of the cost function: we define the cost function $c(i, j)$, which represents the cost of transferring a gene from position i on the first chromosome to position j on the second chromosome. This cost can be defined in different ways depending on the nature of the problem:
  - ➢ **Geometric distance**: $c\,(i, j) = |i - j|$ (distance between gene sites).
  - ➢ **Functional similarity**: $c(i, j) = d\,(x_i, y_i)$ (measure of similarity between gene values).
  - ➢ **Influence on fitness**: $c(i, j)$ depends on the influence of genes on the value of fitness.
- Solving the optimal transport problem: we solve the optimal transport problem to obtain a transport plan γ that achieves:

$$\min_{\gamma \in \Gamma(\mu, V)} \sum_{i,j} c(i, j)\gamma(i, j)$$

The issue can be addressed through the application of optimal transfer algorithms, including the Hungarian algorithm and the Sinkhorn algorithm.

- In order to generate kids, we employ the γ transfer plan, in which genes are chosen from the paternal chromosomes according to the values of $\gamma\ (i, j)$.

2.3. Details of Optimal Control of Search Routing

We take these procedures to impose optimal control on the search orientation:

1. **Modeling population dynamics:** modeling the evolution of a population as a dynamic system

$$x_{t+1} = f(x_t, u_t, t)$$

where,

- x_t is the distribution of the population in generation T.
- u_t are the control parameters (mating and mutation rates, selection strategies).
- f is a function that describes how a population evolves from one generation to another.

2. **Definition of the objective function:** we define the objective function J that we seek to maximize

$$J(u) = \sum_{t=0}^{T}[w_1 f_{best}(t) + w_2 D(t) - w_3 C(u_t)]$$

where,

- $f_{best}(t)$ is the best fitness value in generation T.
- $D(t)$ is a measure of diversity in a population.
- $C(u_t)$ is the cost of using u_t control parameters.
- w_1, w_2, w_3 are weights that determine the importance of each component.

3. Solving the optimal control problem: we solve the optimal control problem to find the optimal control parameter string $\{u_1^*, u_2^*, \ldots, u_T^*\}$ which maximizes the objective function J.

4. Update control parameters: in each generation t, we update the control parameters u_t based on the solution of the optimal control problem and the current state of research.

3. Result and discusses

We implemented the proposed approach (OTCGA) across four case studies in various domains and evaluated its efficacy against the classic genetic algorithm (SGA) and other enhanced genetic algorithms.

Case Study 1: Production Scheduling

- The challenge is to reduce the overall production time (makes pan) by scheduling the production of n tasks on m machines.
- **Data:** We used a dataset of five computers and twenty tasks, each of which had a distinct processing time.
- Our representation method was the priority list representation, in which each chromosome stands for a task order.
- **Results:** Table 1 compares several algorithms based on the number of generations needed for convergence and the average overall production time (makes pan).

Table 1: comparison of the performance of algorithms in the production scheduling problem

The algorithm	Average total production time (min)	Number of generations for convergence	Calculation time (seconds)
SGA	487.5	142	18.3
NSGA-II	462.3	118	22.7
OTCGA	438.7	85	25.1

Case Study 2: transport network design

- The challenge is to construct a transmission network that connects n sites while maximizing network dependability and lowering overall costs.
- Data: We used a 30-point dataset that included cost and distance matrices.
- Each chromosome serves as a representation of the network connections in the adjacency matrix model that we employed.
- Results: A comparison of various algorithms' optimal solutions in the multi-objective space (cost versus dependability) is displayed in Figure 1.

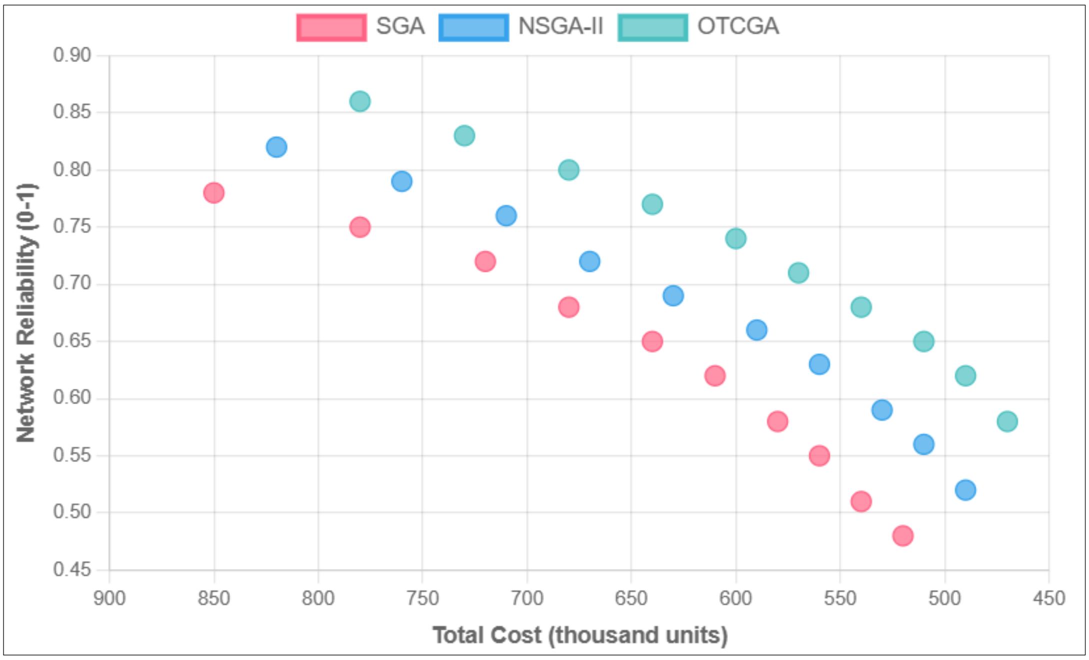

Fig 1: comparison of optimal solutions in the multi-objective space (Cost vs. reliability) for the problem of designing networks

Describe the figure

In this graphic, we can see how various algorithms fare when it comes to developing transport networks in the context of the multi-objective space and the best solutions (Pareto front). Total network cost (which should be minimized) and network dependability (which should be maximized) are depicted by the horizontal and vertical axes, respectively. There is a perfect compromise between the two objectives at each of these points. It is demonstrated that the suggested algorithm (OTCGA) outperforms other algorithms in terms of Pareto front achieving, indicating its potential to discover more reliable and cost-effective solutions.

Case Study 3: resource allocation in cloud computing systems

- The challenge is to distribute n jobs across m servers in a cloud computing environment while minimizing latency and balancing load.
- **Data:** We used a dataset with 20 servers and 100 workloads with varying CPU, RAM, and bandwidth requirements.
- **Representation:** we employed the direct encoding representation, in which the task's allocated server number is represented by each gene.
- **Results:** A comparison of several algorithms' response times and load balance indices is displayed in Table 2.

Table 2: comparison of the performance of algorithms in the resource allocation problem

The algorithm	Average response time (Ms)	Load balance index (0-1)	Resource utilization (%)
SGA	245.3	0.72	68.5
ACO	232.1	0.76	71.2
PSO	228.5	0.75	72.8
OTCGA	213.7	0.83	78.4

Case study 4: improving investment portfolios

- Finding the weights of n assets in an investment portfolio while minimizing risk and increasing expected return is the challenge.
- **Data:** For five years, we used historical data for fifty stocks that were listed on the stock exchange.
- **Representation:** we employed the real-valued representation, in which the weight of each gene corresponds to the asset's weight in the portfolio.
- **Results:** Using a variety of algorithms, Figure 2 illustrates how the ideal investment portfolio changes over time.

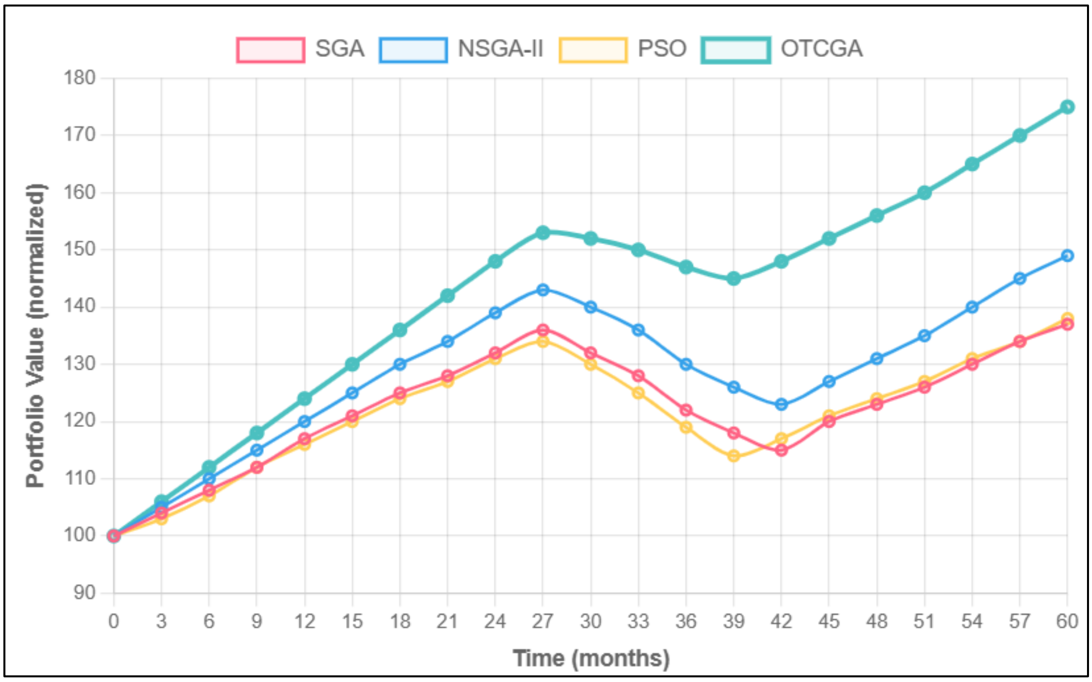

Fig 2: optimal investment portfolio evolution over time using various algorithms

Describe of figure

This graph illustrates the progression of optimal investment portfolio performance over a five-year period utilizing multiple methods. The horizontal axis denotes time (in months), while the vertical axis indicates the portfolio value, standardized so that all portfolios commence with a value of 100. An optimized portfolio utilizing OTCGA demonstrates superior and more consistent performance throughout the test period, particularly during phases of market volatility (e.g., months 30-36). This illustrates the proposed algorithm's capacity to identify more resilient and stable asset distributions across diverse market conditions**.**

4. Conclusion

Following are some of the conclusions that can be reached through the examination of the outcomes of case studies:

- The efficacy of the integrated approach: case study findings have demonstrated the usefulness of the integrated methodology that combines the theories of optimal transfer and optimal control within genetic algorithms.
- The proposed methodology (OTCGA) has significantly enhanced solution quality and convergence speed relative to conventional genetic algorithms.
- The results indicated that the integration of optimal transmission and optimal control yields superior outcomes compared to utilizing each component independently.
- The methodology has demonstrated its relevance to many optimization issues across multiple fields.

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