



Cycles and Affine Subschemes in the Applied Research Process

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Abstract

Can cycles in research refine the research? Yes, cycles in research are essential for refining the research process and its findings. Research is not a linear, one-and-done process but an iterative and recursive cycle of continuous improvement. Each cycle or iteration builds upon the last, allowing researchers to refine their methods, test assumptions, and produce more accurate and reliable result. Then the initial cycle of the commutative diagram of the research method by fundamentals that is isomorphic to a subtheory is such an affine subscheme, which is isomorphic to the spectrum of the ring of the studied research method. A corollary is obtained in the electronics and quantum field theory research.

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1. Introduction

Let be the categories (Verdier, 1996)^[1], (Keller *et al.*, 1996) and (Yekutieli, 2019)^[6], (whose points are sets of propositions) $\Phi_1, \Phi_2, \dots, \Phi_n, \dots$, of an Abelian category^[1] \mathcal{R} . These sets can be set of propositions of certain class α , then these can to define the following category $\mathfrak{I}\Phi_\alpha$, whose objects are sets with arrows and therefore sequences as:

$$\Phi_0 \xrightarrow{d_0} \Phi_1 \xrightarrow{d_1} \Phi_2 \xrightarrow{d_2} \Phi_3 \rightarrow \dots$$

which defines the research process starting of the category of true propositions belonging to a class of the knowledge in mathematics and physics *EFISMAT*, such and as is given in the method by fundamentals (Bulnes, 2010), (Method by Fundaments, Bulnes., 2025). Also, here each Φ_i , is an object of \mathcal{R} , and each compositions $d^{i\circ} d^{i+1}$, is zero. The

i th -cohomology group of the complex is $H^i(\Phi_i) = \ker d^i / \operatorname{im} d^{i-1}$.

If in particular the categories of objects I , *EFISMAT*, $Th\Sigma$, $\mathfrak{I}\Phi_\alpha$, $\{\pi_j\}$, π^* , P , and 0, defined in (Verdier, 1996)^[1], (Keller *et al.*, 1996), (Yekutieli, 2019)^[6], (Bulnes, 2010), and (Method by Fundaments, Bulnes., 2025), we have the research method by fundamentals diagram

¹ This can be a modules category on a ring, or category of sheaves of Abelian groups on a topological space, for example topological groups.

the spectrum isomorphic to the set of integers, which is given by the zero ideal. Likewise, the initial cycle of the commutative diagram of the research method that is isomorphic to a subtheory is such an affine subscheme, since it refers to the structure of the category of modules (or, more specifically, of homomorphisms, remember the homomorphisms $(\{A\})$, that satisfies the integrals in a cycle Γ , in the research method by fundamentals $\oint \Gamma = 0$, (Recurrent Cycles, Bulnes., 2025), where we have applied the Def. 2. 1, Def. 2. 2, and proposition A.1). Indeed, we consider the property of the line integrals on $\Gamma E = \Gamma 1 \cup \Gamma 2 \cup \Gamma 0$, and apply the integration of adjoint cycles considering the diagram (13) of the category's context of the research method by fundamentals with the adequate orientation. Also apply the first theorem of consistence (Harmony and Precision, Bulnes., 2025), (Recurrent Cycles, Bulnes., 2025). of the mathematical theory of the research:

$$\begin{aligned} \dots \rightarrow \mathfrak{I}\Phi_\alpha \rightarrow Th_{App} \rightarrow \{\pi_j\} \rightarrow \pi^* \rightarrow P \rightarrow 0 \\ \searrow \uparrow \mathfrak{D} \Gamma_1 \uparrow \mathfrak{D} \Gamma_2 \uparrow \mathfrak{D} \Gamma_0 \uparrow \nearrow \\ Th\Sigma \rightarrow Th_{App} \rightarrow \{\pi_j\} \rightarrow \pi^* \end{aligned} \quad (14)$$

Then the integral on the cycles satisfies (Recurrent Cycles, Bulnes., 2025), (Cohomology of Cycles, Bulnes., 2008), (Watanabe, 2008, 2009) [5], and (Yekutieli, 2019) [6]:

⁶ A closed subscheme is a fundamental concept in algebraic geometry that refines the notion of a closed subset of a scheme by including a specific "scheme structure" (defined by a sheaf of ideals).

$$\begin{aligned} \oint_{\Gamma E} \Gamma E &= \int_{\Gamma_1 \cup \Gamma_2 \cup \Gamma_0} \phi(Th_{App})\phi(\{\pi_j\}) - \phi(\pi^*)\phi(\{\pi_j\}) \\ &= \int_{\Gamma_1} \phi(Th\Sigma)\phi(Th_{App}) \\ &\quad - \phi(Th_{App})\phi(Th\Sigma) \\ &+ \int_{\Gamma_2} \phi(Th_{App})\phi(\{\pi_j\}) \\ &\quad - \phi(\{\pi_j\})\phi(Th_{App}) + \int_{\Gamma_0} \phi(\{\pi_j\})\phi(\pi^*) \\ &\quad - \phi(\pi^*)\phi(\{\pi_j\}) = 0, \end{aligned} \quad (15)$$

which has values in the category $0\mathfrak{X} \cong \text{Spec}\mathbb{Z}$, [7] which is the prime ideal no vanish which generates a affine subscheme. The key point is that for the ring of integers $\text{Spec}\mathbb{Z}$, the zero ideal is the only non-zero prime ideal and generates a unique affine subscheme that is isomorphic to the set of integers.

Remark: Remember that the cycles represent refinements of research giving integer products. The diagram (14) is an application of the second consistence theorem of the research theory (Harmony and Precision, Bulnes., 2025), (Gelfand, 2003).

⁷ In the category of schemes, $\text{Spec}\mathbb{Z}$, is a terminal object (also known as a final object or a universal base). This means that for any other scheme E , there is a unique morphism (map) from E , to $\text{Spec}\mathbb{Z}$. Here the unique morphism from E , to $\text{Spec}\mathbb{Z}$, is an isomorphism if and only if the scheme E , is affine, which by hypothesis is affine, then E , is isomorphic to $\text{Spec}\mathbb{Z}$.

4. Applications

Some illustrative examples on the applications of the theorem are the following.

Def. 4.1. (Prototype Design). The phrase "spectrum that design the prototype or technology" implies the mathematical framework is being used as a model for an engineered system (e.g., a quantum device or a system with specific resonant properties) which in a context of the research method by fundamentals is identified by π^* .

Proposition 4.1. For $\{\lambda_1, \lambda_2, \dots, \lambda_j\}$, (Simon and Reed, 1972) [7] that are the spectrum that design the prototype or technology. The spectrum degenerates in a harmonics set if the operator $A \in \text{Sym}(H)$, is such that $\forall \psi \in H$ (with H , a Hilbert space) is had that $A\psi = \lambda\psi$, and $\text{Spec}\mathfrak{I}\Phi_\alpha \cong \mathbb{Z}(k)$. Proof. The set of eigenvalues $\{\lambda_1, \lambda_2, \dots, \lambda_j\}$, is an ordered set which is related to the cyclic group of order k , which implies periodically or specific discrete values. Then the set degenerates in harmonics set, which means that the set of eigenvalues typically represents energy levels then the spectrum of some operator $A \in \text{Sym}(H)$, is isomorphic to the set that relates to a topological invariant or symmetry constraint that establish the spectrum type defined by ⁷ In the category of schemes, $\text{Spec}\mathbb{Z}$, is a terminal object (also known as a final object or a universal base). This means that for any other scheme E , there is a unique morphism (map) from E , to $\text{Spec}\mathbb{Z}$. Here the unique morphism from E , to $\text{Spec}\mathbb{Z}$, is an isomorphism if and only if the scheme E , is affine, which by hypothesis is affine, then E , is isomorphic to $\text{Spec}\mathfrak{I}\Phi_\alpha$, to the investigated system. Due that the cyclic group of order k , is the set $\mathbb{Z}(k)$, then $\text{Spec}\mathfrak{I}\Phi_\alpha \cong \mathbb{Z}(k)$.

Example 4.1. Super-integrable quantum systems often hide "accidental degeneracy" due to hidden symmetries, which leads to highly degenerate spectra.

Example 4.2. In electronics research the set of eigenvalues $\{\lambda_1, \lambda_2, \dots, \lambda_j\}$, is an ordered set which is related to the cyclic group of order k , which implies periodically or specific discrete values called harmonics of a fundamental frequency (Bulnes, 1998) [9].

Remark. The second example can be extended a non-cyclic group like $\mathbb{Q}(k)$.

5. Conclusions

The property of the commutative scheme of derived categories of the research method by fundamentals diagram establishes subtheories $SubTh$, whose cycle is such affine subscheme, since it refers to the structure of the category of modules (or, more specifically, of homomorphisms, which by recurrent cycles satisfies the integral (15). Then this has values in the category $0\mathfrak{X} \cong \text{Spec}\mathbb{Z}$, which is the prime ideal no vanish which generates the affine subscheme mentioned. The key point is that for the ring of integers $\text{Spec}\mathbb{Z}$, the zero ideal is the only non-zero prime ideal and generates a unique affine subscheme that is isomorphic to the set of integers.

Remember that the unique morphism is a map from a scheme to the spectrum of its global sections, which is an isomorphism only in the specific case where the scheme is affine.

Likewise, is had the solution $P \rightarrow 0$, that finally gives solution to one society necessity. Finally, this has that see with the design of a prototype in π^* , which is the best prototype to be a product $P \rightarrow 0$. In electronics and quantum field theory research this means that the harmonics set $\mathfrak{X}\Phi_\alpha$, derived from the proper research, its spectrum of some operator $A \in \text{Sym}(H)$, is isomorphic to the set that relates to a topological invariant or symmetry constraint that establish the spectrum type and due to that the cyclic group of order k , is the set $\mathbb{Z}(k)$, then $\text{Spec}\mathfrak{X}\Phi_\alpha \cong \mathbb{Z}(k)$, which is an immediate corollary of the theorem 3.1.

Appendix A. Basic Definitions and Relations

Def. A.1. A technologicism is a neologism that obtains a technology from a technology given. Likewise, $\phi_v(t\gamma) = t\delta$.

Def. A.2. A prototype π , is a technology under research (non- finished product). The theory that generates is a sub-theory or the category *subth* Φ . An optimal prototype π^* , is a technology ready to be product P .

Proposition A.1. ^[6,7]: An applied theory is a sub-theory, therefore *SubTh* $\Phi \cong Thapp$.

All research unit *Cn* Φ_α , is defined as the place (space) where only exist true propositions. Then a theory based on models of true propositions sets is the category of morphisms given to $\text{Spec}\mathbb{Z}$. Remember that the unique morphism is a map from a scheme to the spectrum of its global sections, which is an isomorphism only in the specific case where the scheme is affine. by *Hom* (*ThMod* Φ_α (*EFISMAT*), *Cn* Φ_α) is isomorphic to the set of morphisms (creation of technologies) *Hom* ($\phi_\sigma(t_\gamma), t_\eta$).

Def. A.3. The space $\Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta)$, defines the set of technology transferences of the class τ , to the class σ , formally:

$$\Lambda_{SYSTEM}(\Phi_\alpha(E), t_\delta) = \{\tau | \tau \Rightarrow \sigma\}, \quad (A.1)$$

Def. A.4. The category *Cn* Φ , is the set of tautological applications of the true proposition category Φ , is to say

$$Cn\Phi = \{\sigma | \Phi \vDash \sigma\}, \quad (A.2)$$

This category is called research unit.

Technical notation.

E_{FISMAT} – Derived category of all knowledge in mathematics and physics. Locally is a Banach space ^[15], for example in the technology applications.

Π – Set of prototypes. This set is a derived category whose morphisms are $\{\pi_i\} \rightarrow \pi^*$.

α –Scientific technologicalism or simply technologicalism. This is a homomorphism on the the group of technologies, that is to say, $\forall t_\gamma$, and $t_\delta \in G(\circ)$, $\phi_\alpha(t_\gamma \circ t_\delta) = \phi_\alpha(t_\gamma) \cdot \phi_\alpha(t_\delta)$, where the image $\phi_\alpha(t_\gamma \circ t_\delta) \in G$.

t_α –Technology. Are the points belonging to the group G . $\Phi_\alpha(E)$ –Set of true propositions (useful propositions) in the

creation of a theory. This is a sub-category such that $\Phi_\alpha(E) \subset E_{FISMAT}$. Locally also can be a Banach space.

κ_α –The class α . This sub-category belongs to a partition of the groups G .

Λ_{SYSTEM} –Derived category of systems of utilities, manufactures and tools. Locally also is a Banach space.

Hom (A, B) –Space of homomorphism from the set A . until set B . In a commutative or noncommutative ring is a derived category of functors *Hom*.

ThMod Φ_α –Derived category of all the statements that are true in all models of $\Phi_\alpha(E)$.

\rightarrow –Morphism.

Th Σ – Derived category of all theories on the set of demonstrated and verified proposition in the specific research. This category is the most important in the research method by fundaments.

Cn Φ – Derived sub-category of all consequences by the research realized as final products.

\cong –Isomorphism (structural equivalence between categories, rings, groups, any sets of elements). Another similar notation in the diagrams and schemes is \Downarrow . Also, *Isom*, in the context of a derived category of functors *Hom*. *AffSch* –Category of the affine schemes.

\circlearrowleft –Loops

6. References

- Verdier JL. Des catégories dérivées des catégories abéliennes. Astérisque. 1996;(239):1-253. Paris: Société Mathématique de France. ISSN 0303-1179.
- Thrift H, Musk M. Short reviews of the Francisco Bulnes’s mathematical research theory. Int J Syst Sci Appl Math. 2019;4(1):13-17. doi:10.11648/j.ijssam.20190401.12
- Demaine ED, Demaine ML, Uehara R, Uno Y, Winslow A. Packing cube nets into rectangles with O(1) holes. In: Akiyama J, Marcelo RM, Ruiz MJP, Uno Y, editors. Discrete and computational geometry, graphs, and games. JCDCGGG 2018. Lecture notes in computer science. Vol. 13034. Cham: Springer; 2021. p. 152-64. doi:10.1007/978-3-030-90048-9_12
- Bulnes F. Cohomology of cycles and integral topology. In: Bulnes F, editor. Proc. Meeting of 27-29 Autumn 2008, Mexico City, Internal. Meeting Appliedmath 4, IM-UNAM, Mexico; 2008. Available from: www.Appliedmath4.ipn.mx
- Watanabe T. On Kontsevich’s characteristic classes for higher dimensional sphere bundles I: the simplest class. Math Z. 2009;262:683-712. doi:10.1007/s00209-008-0396-4
- Yekutieli A. Derived categories. Cambridge studies in advanced mathematics. Vol. 183. Cambridge: Cambridge University Press; 2019.
- Gelfand SI, Manin YI. Methods of homological algebra. 2nd ed. Berlin: Springer; 2003.
- Reed M, Simon B. Methods of modern mathematical physics. Vol. I: Functional analysis. New York: Academic Press; 1972. (Note: The provided citation appears to refer to this standard work; the title "Mathematical Methods in Physics" is commonly used interchangeably.)
- Bulnes F. Tratado de matemáticas superiores: análisis de sistemas y señales. México: Facultad de Ciencias,

UNAM, ETC; 1998.

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