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Kantowski-Sachs Cosmological Model with Lyra Geometry in $f(R, T)$ Gravity

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Abstract

In this paper, we study Kantowski-Sachs cosmological model in Lyra geometry based on $f(R, T)$ theory. We considered here $f(R, T) = f_1(R) + f_2(T)$. The field equations are solved by taking θ proportional to σ which gives $A = B^n$, A and B are the metric coefficients. Model's physical behaviors have been studied using some physical parameters. Also, pressure (\tilde{p}), energy density (ρ) and function of Ricci scalar (\tilde{R}) is also evaluated.

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1. Introduction

Universe accelerated expansion confirmed by observations of Type Ia supernovae, cosmic microwave background radiation, and large-scale structure surveys, has motivated the study of modified gravity theory as alternatives to Einstein's General Relativity^[1-3]. Among these theories, $f(R)$ gravity has received considerable attention due to its ability to explain late-time cosmic acceleration without invoking dark energy^[4-5]. Further generalization led to the formulation of $f(R, T)$ gravity, introducing matter-geometry coupling effects^[6].

Anisotropic models play a crucial role in understanding the early stages of the cosmos. Kantowski-Sachs space-time, originally proposed to describe homogeneous but anisotropic cosmologies, has been extensively studied in both General Relativity and modified gravity theories^[7-9]. Such models are useful in explaining anisotropic expansion, black hole interiors, and cosmological singularities^[10].

Another important geometrical modification in gravity was proposed by Lyra, who introduced a gauge function into Riemannian geometry, leading to what is now known as Lyra geometry^[11]. Unlike Weyl geometry, Lyra geometry preserves the integrability of length transfer and introduces a displacement vector field that plays a role analogous to a cosmological constant^[12-13]. Sen and Dunn developed field equations in Lyra geometry which were later applied to various cosmological models^[14-15].

Several authors have explored cosmological solutions of Bianchi and Kantowski-Sachs space-times in Lyra geometry and studied the influence of it on universe's dynamics^[16-17]. More recently, modified gravity theories combined with Lyra geometry have attracted interest, as they provide richer cosmological behavior and improved consistency with observations^[18-22].

The $f(R, T)$ theory of gravity, introduced by Harko et al., allows for explicit dependence of the gravitational action on matter content, leading to non-conservation of the energy-momentum tensor and novel cosmological implications^[23]. Exact solutions in $f(R, T)$ gravity have been obtained for various anisotropic models, including Bianchi and Kantowski-Sachs universes^[24-26]. Motivated by these developments, the present work aims to study a Kantowski-Sachs model in $f(R, T)$ under Lyra geometry. And assuming source as perfect fluid, we derive exact solutions using a physically motivated condition relating to θ and σ . Models kinematical and physical properties are analyzed in details.

Lyra's geometry and modified $f(R, T)$ gravity

Lyra geometry is based on the modification of Riemannian geometry which closely resembles to Weyl's geometry. In this Lyra geometry, Lyra proposed a displacement vector between to neighboring points and its components is $A = dx^i$. The coordinates x^i and the gauge function A constitute the reference frame (A, x^i) .

The curvature tensor of Lyra geometry in the Riemannian geometry and is given by

$$\tilde{R}^i_{j\rho\sigma} = A^{-2} \left[\frac{\partial}{\partial x^\rho} (A\tilde{\Gamma}^i_{j\sigma}) - \frac{\partial}{\partial x^\sigma} (A\tilde{\Gamma}^i_{j\rho}) + A^2 (\tilde{\Gamma}^i_{\lambda\rho}\tilde{\Gamma}^{\lambda}_{j\sigma} - \tilde{\Gamma}^i_{\lambda\sigma}\tilde{\Gamma}^{\lambda}_{j\rho}) \right] \tag{1}$$

Where $\tilde{\Gamma}^i_{j\sigma} = \tilde{\Gamma}^i_{j\sigma} - \frac{1}{2}\delta^i_j\varphi_\sigma$

Here length of the vector is conserved upon parallel transports.

Then, the curvature scalar is

$$\tilde{R} = A^{-2}R + 3A^{-1}\nabla_i\varphi^i + \frac{3}{2}\varphi^i\varphi_i + 2A^{-1}(\log A^2)_{,i}\varphi^i \tag{2}$$

Where $\varphi^i = g^{ij}\varphi_j$ is called displacement vectors field, $R =$ Riemannian curvature scalar.

The action for $f(R, T)$ using Lyra geometry is

$$S = \frac{1}{16\pi G} \int f(\tilde{R}, T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x, \tag{3}$$

Using $A = 1$ and $L = \tilde{R}$, we obtain

$$\tilde{R} = R + 3\nabla_i\varphi^i + \frac{3}{2}\varphi^i\varphi_i \tag{4}$$

Where $\tilde{R} =$ function the Ricci scalar, $T =$ the trace of stress-energy tensor of matter, $g =$ determinant of g_{ij} $L_m =$ matter Lagrangian density and T_{ij} is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}, \tag{5}$$

$$\& T = g^{ij}T_{ij}.$$

Assuming L_m depends only on the components of g_{ij} , we obtain

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}}, \tag{6}$$

Varying the action with respect to g_{ij} , we get,

$$f_{\tilde{R}}(\tilde{R}, T)\tilde{R}_{ij} - \frac{1}{2}f(\tilde{R}, T)g_{ij} - (\nabla_i\nabla_j - g_{ij}\square)f_{\tilde{R}}(\tilde{R}, T) = -\frac{8\pi G}{c^2}T_{ij} - f_T(\tilde{R}, T)(T_{ij} + \theta_{ij}), \tag{7}$$

Where $\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2}{\partial g^{ij}\partial g^{lm}}$ (8)

Here $f_{\tilde{R}}(\tilde{R}, T) = \frac{\partial f(\tilde{R}, T)}{\partial \tilde{R}}$, $f_T(\tilde{R}, T) = \frac{\partial f(\tilde{R}, T)}{\partial T}$, $\square \equiv \nabla^i\nabla_i$ Where ∇_i is covariant derivatives.

Assume perfect fluid as source matter, T_{ij} is derived as

$$T_{ij} = (\rho + \tilde{p})u_iu_j - \tilde{p}g_{ij}, \tag{9}$$

The perfect fluids is described by ρ, p & $u^i = (0,0,0,1)$ in moving co-ordinate system satisfying $u_iu^i = 1$ and $u_i\nabla_ju^i = 0$.

We can assume $L_m = -\tilde{p}$, which gives

$$\theta_{ij} = -2T_{ij} - \tilde{p}g_{ij}, \tag{10}$$

Harko et. al. considers three functional form of $f(R, T)$.

In this paper, we considered

$$f(\tilde{R}, T) = f_1(\tilde{R}) + f_2(T) \tag{11}$$

Where $f(\tilde{R}, T)$ is function of traces of the stress tensor.

Using Eqns (11), Eqns (7) becomes

$$f_{\tilde{R}}(\tilde{R}, T)\tilde{R}_{ij} - \frac{1}{2}f_1(\tilde{R}, T)g_{ij} - (\nabla_i\nabla_j - g_{ij}\square)f'_{\tilde{R}}(\tilde{R}, T) = -\frac{8\pi G}{c^2}T_{ij} + f_2'(T)T_{ij} + \left[f_2'(T)p + \frac{1}{2}f_2(T) \right]g_{ij} \tag{12}$$

In this case $f(\tilde{R}, T)$ gravity field equation for perfect fluid matter by assuming $f_1 = \lambda\tilde{R}$ and $f_2 = \lambda T$ where λ is taken as arbitrary constant.

With this condition equations (12), reduce to

$$\tilde{R}_{ij} - \frac{1}{2}\tilde{R}g_{ij} = -\left(\frac{8\pi G - \lambda c^2}{c^2} \right)T_{ij} + \left[\tilde{p} + \frac{1}{2}T \right]g_{ij} \tag{13}$$

Using Eqns (4) in (13), we obtain

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_i\phi^i = -hT_{ij} + \left[\tilde{p} + \frac{1}{2}T \right]g_{ij} \tag{14}$$

Where $\phi^j = (0,0,0, \beta(t))$ is displacement vectors field and $h = \frac{8\pi G - \lambda c^2}{\lambda c^2}$ is called scale factor.

Metric and the Field Equations

Kantowski-Sachs space time is

$$ds^2 = dt^2 - A^2(t)dr^2 - B^2(t)[d\theta^2 - \sin^2\theta d\varphi^2] \tag{15}$$

Here $A = A(t)$ & $B = B(t)$.

Kantowski-Sachs Ricci scalar is

$$R = \frac{2\dot{A}}{A} + \frac{4\dot{B}}{B} + \frac{4A\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2} + \frac{2}{B^2} \tag{16}$$

Where dot (\bullet) denotes with respect to t .

Solving Eqns (14) with (9) & (15) gives

$$\frac{2A\dot{B}}{AB} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} + \frac{3}{4}\beta^2 = h\rho - \frac{1}{2}[\rho - \tilde{p}] \tag{17}$$

$$\frac{\dot{B}}{B} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} + \frac{3}{4}\beta^2 = -h\tilde{p} - \frac{1}{2}[\rho - \tilde{p}] \tag{18}$$

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{A\dot{B}}{AB} + \frac{3}{4}\beta^2 = -h\tilde{p} - \frac{1}{2}[\rho - \tilde{p}] \tag{19}$$

Kantowski-Sachs Solution of field equation

Subtracting Eqns (17) from Eqns(18),we get

$$\frac{\ddot{B}}{B} - \frac{2\dot{A}\dot{B}}{AB} = -h(\rho + \tilde{p}) \tag{20}$$

Similarly Subtract Eqns(17) from Eqns(19),we get

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{1}{B^2} - \frac{\dot{B}^2}{B^2} = -h(\rho + \tilde{p}) \tag{21}$$

Here, we have two Eqns(20) and Eqns(21) with four unknowns A, B, \tilde{p} & ρ .

Now to obtain a definite solution, we use $\theta \propto \sigma$ which gives

$$A = B^n \tag{22}$$

By Solving Eqns (20) and Eqns (21), we get independent equation is given as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{1}{B^2} - \frac{\dot{B}^2}{B^2} = 0 \tag{23}$$

Inserting $A = B^n$ in Eqns (23), we get

$$\ddot{B} + \frac{(n^2-1)}{nB} \dot{B}^2 = \frac{1}{nB} \tag{24}$$

Now we take $g(B) = \dot{B}$, we get

$$\frac{dg^2}{dB} + \frac{2(n^2-1)}{nB} g^2 = \frac{1}{nB} \tag{25}$$

Which is linear differential equations.

$g = \sqrt{\frac{1}{2(n^2-1)} + k_1}$ where k_1 is constant of integration

Solving Eqns (25), we obtain

$$A = \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{\frac{n}{2}} t^n + k_2$$

$$B = \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{\frac{1}{2}} t + k_2$$

Where k_2 is constant of integrations

Without loss of generality, we can take $k_2 = 0$, value of A and B , becomes

$$A = \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{\frac{n}{2}} t^n$$

$$B = \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{\frac{1}{2}} t \tag{26}$$

Putting the value of scalar factor A and B in Eqns (15), Kantowski Sachs Solution becomes

$$ds^2 = dt^2 - \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^n t^{2n} dr^2 - \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right) t^2 d\theta^2 + \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right) t^2 \sin^2 \theta d\varphi^2 \tag{27}$$

Physical and Kinematical Character of the Model

Adding Eqns(17) and Eqns(18),we get displacement vector

$$\beta^2 = -\frac{4}{3} \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{-1} t^{-2} - \frac{4(n+1)}{3} t^{-2} \tag{28}$$

Using Eqns(18) and Eqns (20),we obtain pressure and energy density as

$$\tilde{p} = \frac{2n}{t^2} + 2 \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{-1} t^{-2} \tag{29}$$

$$\rho = \frac{1-n^2}{t^2} - \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{-1} t^{-2} \tag{30}$$

Function of Ricci scalar (\tilde{R}) is

$$\tilde{R} = 2t^{-2} \left[(n^2 + n + 1) + \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{-1} \right] + \frac{3}{2} \beta^2 \tag{31}$$

The $f(\tilde{R}, T)$ gravity is

$$f(\tilde{R}, T) = \lambda \left[\frac{(n^2-4n-1)}{t^2} + \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{-1} \frac{1}{t^2} + \frac{3}{2} \beta^2 \right] \tag{32}$$

The spatial volume V and the scale factor a are given by

$$V = \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{\frac{n+2}{2}} t^{n+2} \sin \theta \tag{33}$$

$$a = \left(\frac{1+2(n^2-1)k_1}{2(n^2-1)}\right)^{\frac{n+2}{6}} t^{\frac{n+2}{3}} \sin^{\frac{1}{3}} \theta \tag{34}$$

The parameter H and the scalar expansion θ

$$H = \left(\frac{n+2}{3t}\right) \tag{35}$$

$$\theta = \frac{n+2}{t} \tag{36}$$

The shear scalar σ^2

$$\sigma^2 = \frac{(n-1)^2}{3t^2} \tag{37}$$

Summary and Conclusion

We studied kantowski-sachs cosmological model in Lyra geometry based on $\theta = g^{ij} \theta_{ij} = \theta_i^i$. gravity, Here perfect fluid is taken as source of matter. A linear form $f(R, T) = f_1(R) + f_2(T)$ proposed by Harko et.al. is used. kantowski-sachs solution of field equation are solved by taking expansion scalar θ proportional to the shear scalar σ which gives $A = B^n$. Equation of spatial volume, indicate that universe is expanding. Generalized Hubble parameter has finite positive value, this shows that universe is expanding and as time increases, expansion rate slows and tends to zero when time tends to infinity.

This shows that universe expansion rate is fast during early period and slowing as time increases. From equation of shear scalar, it is seen that, shear scalar has finite value at initial stage and it is tending to zero as time approaches to infinity. This indicates that universe anisotropy is tending towards isotropy over time. Equation for expansion scalar θ which measure the rate of expansion tends to infinity for $t \rightarrow 0$ and vanishes for $t \rightarrow \infty$.

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