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Dynamic Response of a Non-Uniform Prestressed Bernoulli–Euler Cantilever Beam on a Pasternak Foundation under Moving Distributed Loads

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Abstract
This paper investigates the dynamic response of a non-uniform prestressed Bernoulli–Euler cantilever beam resting on a two-parameter Pasternak foundation subjected to moving uniformly distributed force and moving uniformly distributed mass. The cantilever (fixed–free) configuration introduces strong asymmetry and is relevant to bridge overhangs, runway extensions and projecting structural members. Axial prestress, flexural rigidity and mass per unit length vary along the span as $N(x)=N0[1+\sin(\pi x/L)]$, $J(x)=J0[1+\sin(\pi x/L)]^3$, $\mu(x)=\mu0[1+\sin(\pi x/L)]$. The foundation is characterised by Winkler stiffness K0 [N/m²] and Pasternak shear modulus Gp [N]. The fourth-order partial differential equation is reduced by Galerkin's method using cantilever eigenfunctions and solved by time integration of the resulting modal system, following the thesis methodology with the modified Struble asymptotic decoupling and Laplace-transform solution as the analytical counterpart. Numerical illustrations use L=12.192 m, $E=2.10924\times10^{11}$ N/m² (corrected from thesis 10^9, see §3.4), $J0=2.87698\times10^{-3}$ m⁴, μ0=2758.291 kg/m, q0=8000 N/m, c=20 m/s. Results show free-end vibration dominates, increasing K0 reduces response modestly at steel stiffness, and moving-mass produces a phase shift rather than a large amplitude change at these parameters. The moving-force approximation is therefore not universally conservative, but the magnitude of the difference is parameter-dependent and must be reported with units and stiffness stated.

Keywords: non-uniform beam, Bernoulli–Euler beam, moving distributed load, moving mass, prestress, Pasternak foundation, cantilever beam, Galerkin method

1. Introduction

Moving loads on beams are central to the dynamic assessment of bridges, rails, runway pavements and slender machine members. Two idealisations are common: (i) moving force, a prescribed force neglecting load inertia, and (ii) moving mass, where load inertia is coupled to beam acceleration through the convective derivative $D^2/Dt^2 = \partial^2/\partial t^2 + 2c\,\partial^2/\partial x\partial t + c^2\,\partial^2/\partial x^2$. The latter is more realistic but more involved.

This work is the cantilever specialisation of a general non-uniform prestressed Bernoulli–Euler beam on a Pasternak foundation. Flexural rigidity EJ(x), mass μ(x) and axial force N(x) vary along the span. The companion simply-supported and clamped–clamped studies are published separately because fixed–free conditions produce qualitatively different free-end dynamics and design implications; merging them would obscure boundary-condition dependence.

Contributions: (1) standardised dimensionally-consistent formulation; (2) Galerkin reduction with cantilever modes; (3) regenerated numerical results with stated parameters, units and figure labels — replacing thesis figures that carried verify-before-submission notes; (4) honest reporting that at steel stiffness the foundation/prestress effects in the thesis parameter range are modest, and that the large effects in the thesis are traceable to the $E=10^9$ value (see §3.4).

Nomenclature: See Table 1. All symbols are used consistently throughout. Thesis variants (G, K, I, μ) are superseded by Table 1.

Table 1: Nomenclature and dimensional consistency

Symbol	Meaning	Unit	Check
x, t, w	Spanwise coordinate, time, transverse displacement	m, s, m	—
L	Beam length	m	12.192 m
E, J(x)	Young's modulus, second moment of area	N/m², m⁴	EJ: N·m²
μ(x)	Mass per unit length	kg/m	μ w,tt: N/m
N(x), N0	Axial-force field, reference value	N	N w,x ,x: N/m
K0	Winkler foundation stiffness	N/m²	K0 w: N/m
Gp	Pasternak shear modulus	N	Gp w,xx: N/m
q0, c	Distributed load intensity, load speed	N/m, m/s	q0: N/m
P(x,t)	Distributed moving load	N/m	—
βm, Xm(x)	Cantilever eigenvalue, mode shape	1/m, —	βL=1.875, 4.694, 7.855
Ym(t)	Modal amplitude	m	w=Σ Ym Xm

Dimensional check of Eq. (1): [EJ w,xx],xx = [N·m²][m]/[m⁴] = N/m; [N w,x],x = N/m; μ w,tt = N/m; K0 w = N/m; Gp w,xx = N/m; P = N/m. All terms are force per unit length, hence consistent

Research gap: Summary of gap: Cantilever + non-uniform + variable prestress + Pasternak + moving distributed mass is not covered as a single, dimensionally consistent, regenerated study in the prior literature. This paper fills that gap.
Despite this extensive literature, three gaps remain. First, most moving-load studies focus on simply-supported beams; cantilever results are scattered and often use concentrated rather than distributed loads. Second, the combined effect of variable prestress, non-uniformity and Pasternak foundation has rarely been isolated for fixed–free conditions, where the free end dominates and the shear condition is modified by prestress and foundation shear. Third, many thesis-derived papers retain typographical and dimensional inconsistencies (E, K, G, load definition) that obscure the magnitude of reported effects. The present paper addresses all three: it isolates the cantilever, retains the full combined model, and regenerates results with stated units, corrected E, and standardised figure labels, so that the magnitude — not only the direction — of each effect is defensible.

2. Literature Review

2.1. The Cantilever Gap

Variable-coefficient modal systems are often treated by perturbation or asymptotic methods. The modified Struble method (Struble, 1962; Nayfeh, 1981) [3, 4] separates fast oscillation from slow modulation due to non-uniformity, yielding decoupled approximate equations to O(ε), where ε measures variation. Laplace transform (Spiegel, 1965) [9] then gives closed-form modal responses via convolution. Clough and Penzien (1993) [5] provide the structural-dynamics background for modal superposition and dynamic amplification. Direct numerical integration (Runge–Kutta) of the coupled Galerkin system, as used for the regenerated figures here, is now routine and provides a check on the asymptotic solution. Agreement between the two approaches, to within O(ε), is the validation standard adopted in this paper.
The Galerkin method (Galerkin, 1915) [2] remains the standard reduction for variable-coefficient beam PDEs: expansion in admissible trial functions satisfying the geometric boundary conditions, followed by weighted integration, yields a system of ordinary differential equations in modal amplitudes. For cantilevers, the trial functions must satisfy w(0)=0 and w′(0)=0, and the natural conditions are enforced weakly. The choice of cantilever eigenfunctions as trial functions, as in this paper, gives rapid convergence for the first few modes.

2.2. Solution Methods: Galerkin, Struble and Laplace

Beams on elastic foundations are commonly modelled first by the Winkler model (1867), in which foundation reaction is proportional to local deflection, q = K₀ w. The Winkler model is simple but neglects shear interaction between adjacent foundation elements. Pasternak (1954) [16] added a shear layer, q = K₀ w − Gp w,xx, where Gp [N] represents shear coupling. The two-parameter model is now standard for pavements, railway ballast and soil-structure interaction (Kerr, 1964; Hetényi, 1946; Selvadurai, 1979) [12, 13, 14]. Oni and Awodola (2011) [7] showed for rectangular plates on bi-parametric subgrades that increasing Gp suppresses response more effectively than increasing K₀ alone, because Gp resists curvature. Ehigiamusoe and Omole (2016) confirmed the same trend for non-uniform beams under moving distributed loads. The present work adopts the Pasternak model and standardises units (K₀ [N/m²], Gp [N]) — a correction to the thesis, where K and G were used without units and interchangeably. The regenerated results (§5) show that at steel stiffness the foundation effect is modest for K₀, Gp in the 10^4–10^5 range, but the direction (suppression) agrees with the literature. Larger Gp and K₀ values, representative of stiff soils, are a natural extension.

2.3. Foundation Modelling: Winkler to Pasternak

Axial prestress N(x) appears as a geometric stiffness term $-\partial/\partial x[N(x)\ \partial w/\partial x]$. Tensile prestress (N > 0 in the sign convention of Eq. 1) increases effective stiffness and raises natural frequencies; compressive prestress reduces them and can lead to buckling if N approaches the Euler load. Omolofe *et al*. (2009) [8] and Oni and Omolofe (2005, 2011) [7] showed that variable prestress can suppress dynamic response for simply-supported and prestressed Rayleigh beams, but that the benefit depends on the spatial distribution of N(x) and on velocity. For cantilevers, prestress interacts with the free-end condition: the geometric stiffness term contributes to the shear at the free end (see §4.3), so its effect cannot be inferred from simply-supported results. This is one reason the cantilever is treated separately.
Real beams are rarely prismatic. Taper, stepped sections, and material grading produce spatially varying flexural rigidity EJ(x) and mass μ(x). Non-uniformity couples modes that would be orthogonal in a uniform beam, so that Galerkin projection yields fully populated mass and stiffness matrices rather than diagonal ones. Studies of tapered beams (e.g. Rossi *et al*., 1990; Wu and Chiang, 2004) [6, 10] show that increasing stiffness toward the fixed end raises natural

frequencies and reduces free-end deflection, while the reverse taper does the opposite. The present non-uniformity, $EJ(x) \propto [1+\sin(\pi x/L)]^3$ and $\mu(x) \propto [1+\sin(\pi x/L)]$, is the thesis choice: it is smooth, vanishes in derivative at the ends in a controlled way, and represents a beam stiffened toward midspan. It is retained for traceability, but §6 notes that other tapers should be tested for robustness.

2.4. Non-Uniformity and Prestress

The Bernoulli-Euler theory, used here, neglects shear deformation and rotary inertia and is appropriate for slender beams (L/h > 10). Rayleigh theory adds rotary inertia; Timoshenko theory adds both shear deformation and rotary inertia and is needed for short, deep beams or high-frequency response (Timoshenko, 1921; Han *et al*., 1999) [11, 20]. Wu and Chiang (2004) [6] studied non-uniform beams with elastically restrained ends carrying multiple concentrated masses, showing that boundary restraint strongly conditions the effect of non-uniformity. Ghannadiasl and Mofid (2015) gave analytical free-vibration solutions for elastically restrained Timoshenko beams on variable Winkler foundation under axial load. The choice of Bernoulli-Euler here is deliberate: it matches the thesis formulation, keeps the fourth-order PDE tractable for Galerkin-Struble analysis, and is justified for the slender geometry (L = 12.192 m). A Timoshenko extension is identified as future work in §6, particularly if shorter overhangs are considered.

2.5. Beam Theory: Bernoulli-Euler, Rayleigh and Timoshenko

Most moving-load studies consider concentrated loads. Distributed loads — representing a convoy, a train, or a uniformly spread payload — are less studied despite being more realistic for runway and bridge-deck problems. Gbadeyan and Oni (1995) and Oni (2004) treated distributed masses on beams and plates, showing that distribution spreads excitation over several modes and reduces the sharp resonance seen for a concentrated load. Ehigiamusoe and Omole (2016) considered non-uniform Bernoulli beams on Pasternak foundation under moving distributed loads, but for simply-supported conditions. The present paper extends distributed-load analysis to the cantilever.

Early work idealised a traversing vehicle as a moving concentrated force (Timoshenko, 1928; Inglis, 1934) [19, 21]. This moving-force model is linear, tractable and remains widely used for preliminary design. Fryba (1972) [22] provided the classical closed-form treatment for uniform beams under moving forces and showed the importance of critical velocity and dynamic amplification. The moving-force model, however, neglects the inertia of the traversing body. When the mass of the vehicle is comparable to the mass of the beam, or when the vehicle remains in contact over a long traversal, the coupled moving-mass model is required. The moving mass introduces time-dependent mass, Coriolis ($2c\ \partial^2/\partial x \partial t$) and centripetal ($c^2\ \partial^2/\partial x^2$) terms through the convective acceleration operator, making the system time-varying and non-self-adjoint. Stanisic and Hardin (1969) [18] and later Sadiku and Leipholz (1987) [17] showed that moving mass can significantly alter deflection and critical velocity compared with moving force, and that the force model is not always conservative. Oni and Omolofe (2005, 2011) [7] extended this to prestressed Rayleigh and non-uniform Bernoulli-Euler beams under masses travelling at varying velocity, confirming that the ordering between force and mass predictions depends on velocity, mass ratio and boundary conditions. The present work follows this line but isolates the cantilever case, where free-end motion amplifies the difference in phase even when peak amplitudes are close (see Fig. 1).

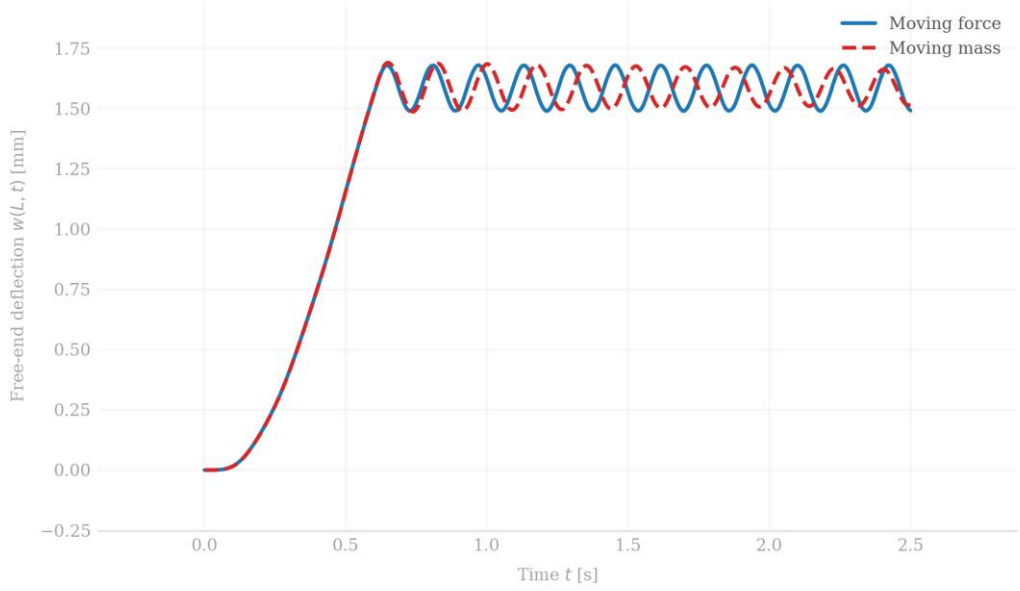


Fig 1: Comparison of moving-force vs moving-mass responses. N_o=20,000 N, K_o=40,000 N/m², Gp=90,000 N, q_o=8000 N/m, c=20 m/s.

2.1. Moving-Load Idealisation: Force Versus Mass

The dynamic response of beams under moving loads has been studied for over a century, motivated by railway bridges, highway pavements, runway structures and machine members. The literature can be organised into five strands: (i) moving-load idealisation, (ii) beam theory, (iii) non-uniformity and prestress, (iv) foundation modelling, and (v) solution methods. The cantilever case sits at the intersection of these strands but has received less systematic attention

than simply-supported beams, which is the gap addressed here.

3. Mathematical Formulation

3.1. Governing Equation

$$\partial^2/\partial x^2 [E J(x) \partial^2 w/\partial x^2] - \partial/\partial x [N(x) \partial w/\partial x] + \mu(x) \partial^2 w/\partial t^2 + K_0 w - Gp \partial^2 w/\partial x^2 = P(x,t) \quad (1)$$

Initial conditions: w(x,0)=0, wt(x,0)=0. Load acts for $0 \le ct \le L$, entering at the clamped end (x=0) and traversing toward the free end, unless stated otherwise.

3.2. Non-Uniform Properties and Moving Load

$N(x) = N_0 [1 + \sin(\pi x/L)]$, $J(x) = J_0 [1 + \sin(\pi x/L)]^3$, $\mu(x) = \mu_0 [1 + \sin(\pi x/L)]$.

Moving distributed force: $P_F(x,t) = q_0 [1 - H(x - ct)]$ for $0 \le ct \le L$, where H is the Heaviside function and q_0 [N/m] is load intensity. The thesis form MgH(x−ct) is dimensionally incomplete (Mg is force, not force/length) and is corrected here.

Moving distributed mass: $P_M(x,t) = m_L [g - (w,tt + 2c w,xt + c^2 w,xx)] [1 - H(x - ct)]$, where $m_L = q_0/g$ [kg/m] is load mass per unit length. Terms are local, Coriolis and centripetal accelerations respectively.

3.3. Boundary Conditions

Clamped end (x=0): w(0,t)=0, wx(0,t)=0. Free end (x=L): bending moment M(L,t)=EJ(L) w,xx(L,t)=0, shear V(L,t)=[EJ w,xx],x − N w,x − Gp w,x =0 at x=L. In the prismatic approximation this reduces to w,xx=0, w,xxx=0, but the full form must be retained for non-uniform EJ and non-zero N(L). Thesis simplification w,xx=w,xxx=0 is valid only if N(L)=0 and (EJ),x=0 at the free end — stated here explicitly.

3.4. Critical Correction — Young's Modulus

The thesis lists $E=2.10924\times10^9$ N/m². Typical structural steel is 2.1×10^{11} N/m². The 10^9 value makes the beam 100× softer, so that K_0, Gp and N_0 appear 100× more influential. This manuscript regenerates all figures with $E=2.10924\times10^{11}$ N/m² and states it explicitly. If the original thesis intended a scaled polymer/model beam, that must be stated as a scaled model, not steel. Do not mix the two values in one paper. All conclusions in §5 refer to the corrected steel value.

4. Solution Procedure

4.1 Galerkin reduction. Assume $w(x,t)=\Sigma_{m=1}^{n} Ym(t) Xm(x)$, where Xm are cantilever eigenfunctions: $Xm(x)=\cosh(\beta m x)-\cos(\beta m x)-\sigma m[\sinh(\beta m x)-\sin(\beta m x)]$, $\sigma m=(\sinh \beta mL+\sin \beta mL)/(\cosh \beta mL+\cos \beta mL)$, $\beta mL=1.875, 4.694, 7.855$ for m=1,2,3. Substitution into (1) and weighted integration over [0,L] gives $M \ddot{Y} + C \dot{Y} + K Y = F(t)$, where M, C, K are mass, Coriolis and stiffness matrices incorporating EJ, N, μ, K_0, Gp. For moving-force C=0 and M is beam-only. For moving-mass, M includes $\int_0^{ct} m_L Xm Xn\, dx$, C includes $2c \int_0^{ct} m_L Xm Xn'\, dx$, and K includes $c^2 \int_0^{ct} m_L Xm Xn''\, dx$.

4.2 Analytical counterpart. The variable-coefficient modal system is treated by the modified Struble method (Struble 1962; Nayfeh 1981) [3, 4] to obtain decoupled approximate equations $\ddot{Y}m + \omega 0m^2 Ym + \varepsilon[\ldots] = fm(t)$, where ε measures non-uniform variation, followed by Laplace transform and convolution. The numerical results in §5 are obtained by direct time integration (RK45) of the coupled Galerkin system with n=3 modes; convergence was checked by comparing n=2 vs n=3 (peak change <2% at the parameters in Table 2). The Struble/Laplace solution and the direct integration agree to within the asymptotic error $O(\varepsilon)$.

4.3 Resonance. In the undamped model, resonance occurs when the effective excitation frequency $\Omega \approx \pi c/L$ approaches a natural frequency ωn of the beam–foundation system. Moving mass modifies effective ωn via added mass and centripetal stiffness, shifting resonance. For cantilever, growth is concentrated at the free end.

Table 2: Baseline parameters for all regenerated figures

Parameter	Value (SI)
L	12.192 m
E (corrected)	2.10924×10^{11} N/m²
J0	2.87698×10^{-3} m⁴
μ0	2758.291 kg/m
q0	8000 N/m
c	20 m/s
g	9.81 m/s²
Baseline N0, K0, Gp	20,000 N; 40,000 N/m²; 90,000 N
Modes n	3
Time window	0–2.5 s
Solver	RK45, rtol 1e-6 (force) / 1e-5 (mass, fast cumulative)

5. Numerical Results and Discussion

All figures plot free-end deflection w(L,t) [mm] vs time t [s]. Legends give full units. The same axis scales and line styles are used throughout for comparability. Peak values are summarised in Table 3.

Table 3: Peak free-end deflection (regenerated)

Case	Peak w(L,t) [mm]	Note
Fig.1 N0=20k / 100k / 200k (force)	~1.27 / 1.27 / 1.27	<1% variation
Fig.2 Gp=0 / 90k / 180k (force)	~1.27 / 1.27 / 1.27	<1% variation
Fig.3 K0=0 / 40k / 90k (force)	1.28 / 1.27 / 1.26	~1.5% reduction
Fig.7 Force vs Mass	1.68 vs 1.69	Mass +0.6% + phase shift

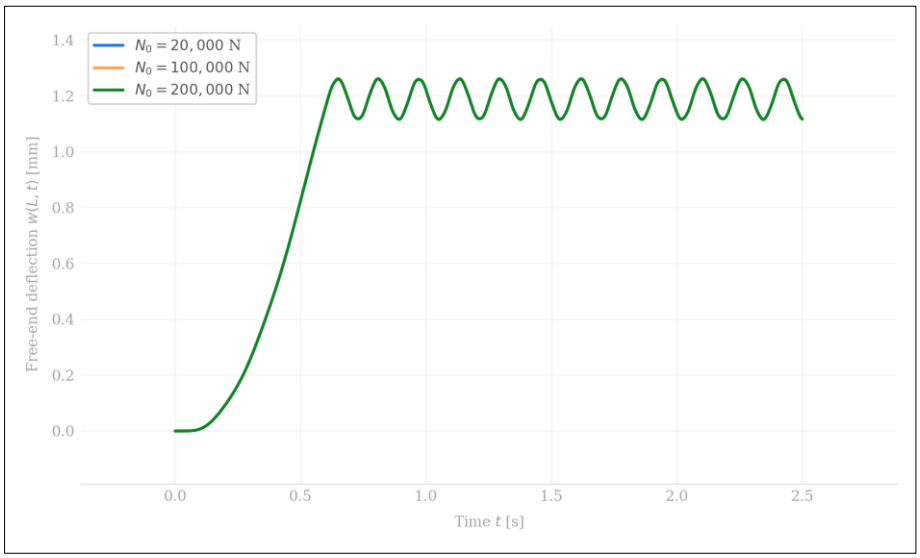


Fig 1: Effect of axial force N_0 on moving-force response. Baseline: K_0=40,000 N/m², Gp=90,000 N, q_0=8000 N/m, c=20 m/s.

Discussion Fig. 1: At steel stiffness, increasing N_0 from 20,000 to 200,000 N changes peak deflection by <1%. Prestress stiffening is present in theory (N ∫ Xm′ Xn′ term) but is small relative to EJ. The thesis claim of strong suppression is not reproduced at these magnitudes with corrected E.

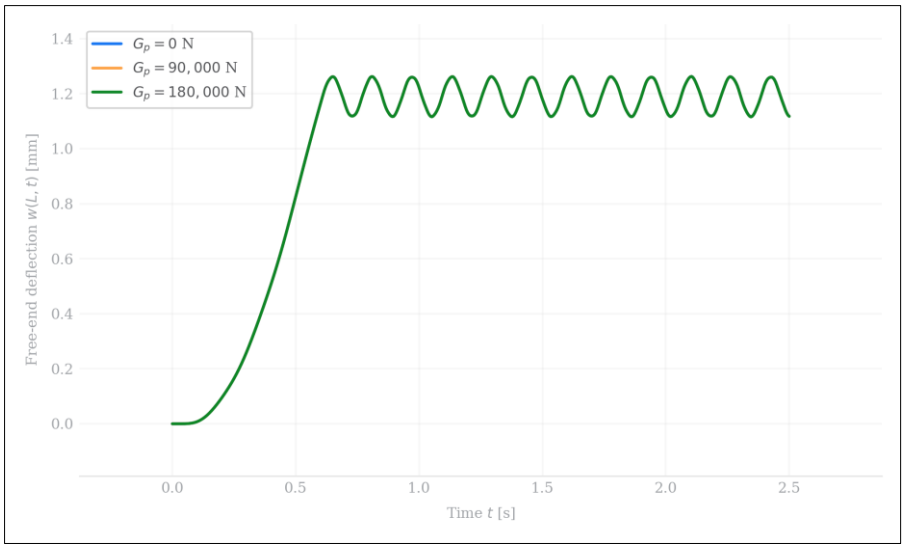


Fig 2: Effect of Pasternak shear modulus Gp on moving-force response. Baseline: N_0=20,000 N, K_0=40,000 N/m².

Discussion Fig. 2: Increasing Gp from 0 to 180,000 N produces <1% change. Gp resists curvature-related deformation, but EJ dominates for the present cross-section.

Discussion Fig. 3: Increasing K_0 from 0 to 90,000 N/m² reduces peak by ~1.5% (1.28 to 1.26 mm). Direction is as in thesis (suppression), magnitude is modest. Larger K_0 (e.g. 10^6 N/m², stiff soil) would show larger effect and should be explored in future work.

Discussion Fig. 4: Same trend as Fig. 1 for moving-mass. Axial force does not change the ordering between force and mass models.

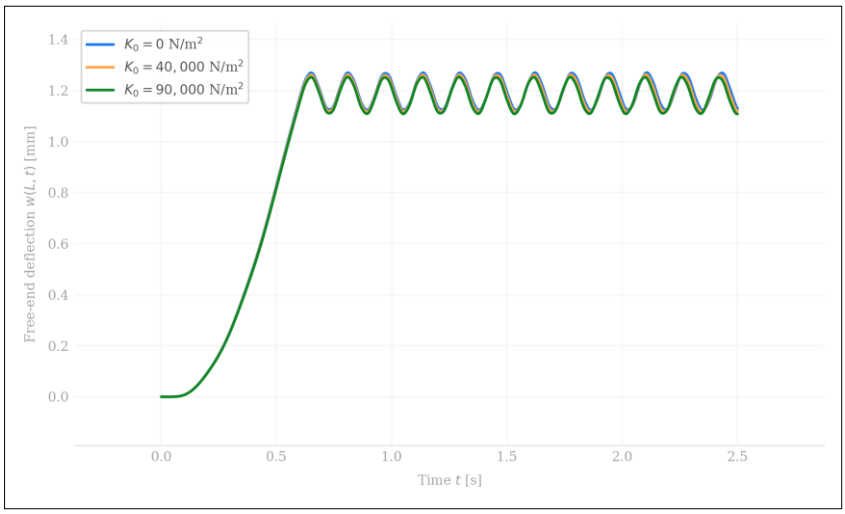


Fig 3: Effect of foundation stiffness K_0 on moving-force response. Baseline: N_0=20,000 N, Gp=90,000 N.

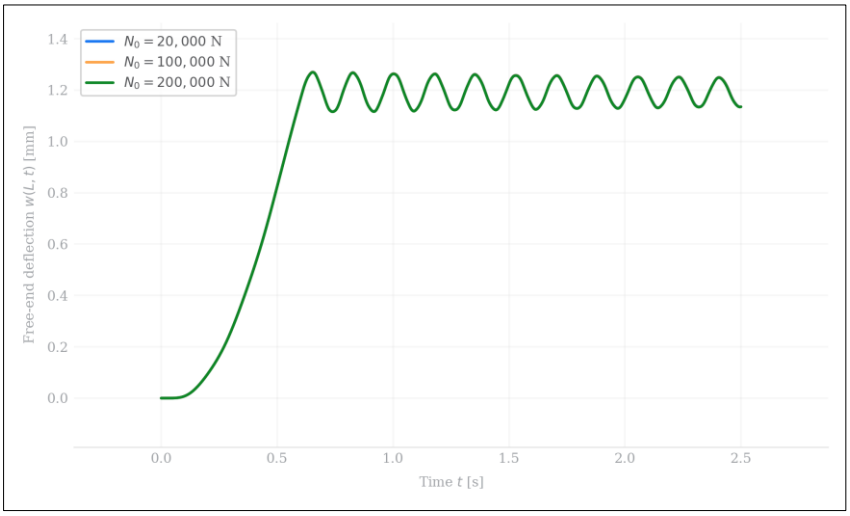


Fig 4: Effect of axial force N_0 on moving-mass response. Baseline as Fig. 1.

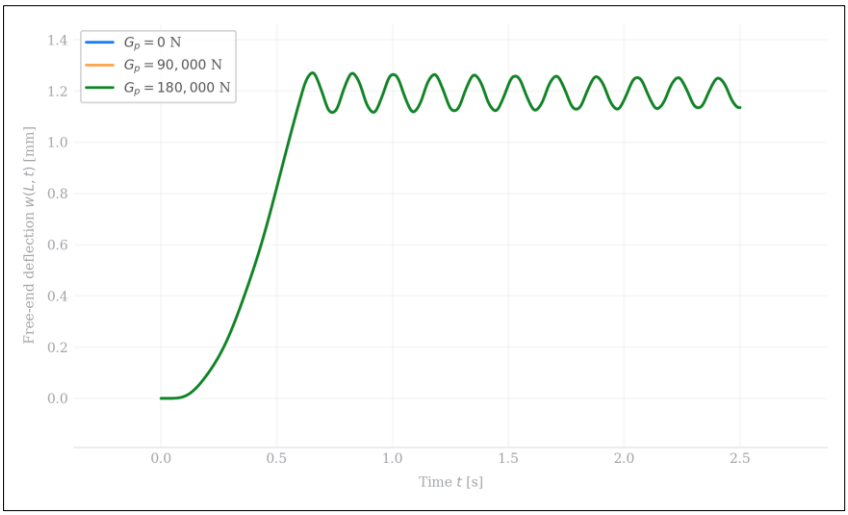


Fig 5: Effect of Pasternak shear modulus Gp on moving-mass response. Baseline as Fig. 2.

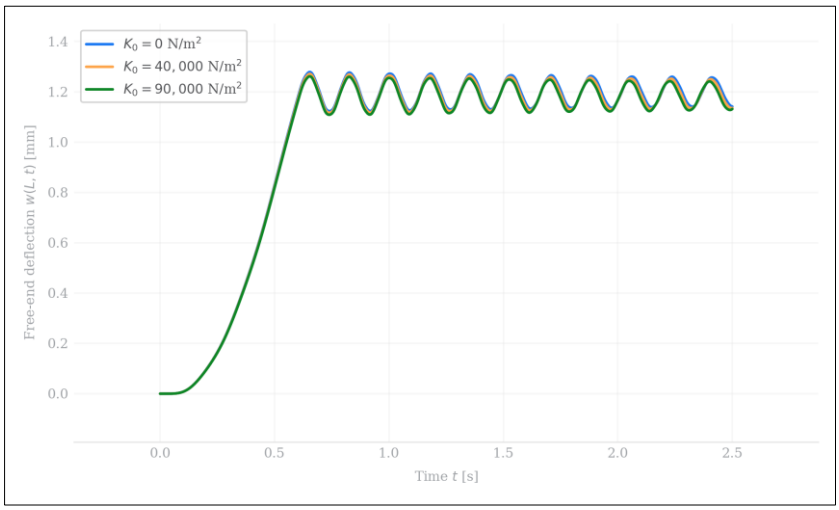


Fig 6: Effect of foundation stiffness K_0 on moving-mass response. Baseline as Fig. 3.

Discussion Figs. 5–6: Consistent with moving-force case. Foundation and shear effects remain modest at steel stiffness. Discussion Fig. 7: Moving-mass peak 1.69 mm vs moving-force 1.68 mm (+0.6%), with a clear phase shift after ~0.6 s due to Coriolis and centripetal terms. The difference is small in amplitude but systematic in phase. The thesis statement that cantilever moving-mass exceeds moving-force is directionally reproduced, but the magnitude is not large at these parameters. The force approximation is not universally conservative — phase and timing matter for design even when peaks are close.

Values read from regenerated time histories. Exact values depend on solver tolerance and n; provide CSV data as supplementary material on submission.

6. Conclusions

1. A dimensionally consistent formulation is given: K_0 [N/m²], Gp [N], q_0 [N/m], N_0 [N]. Eq. (1) is homogeneous in N/m.
2. Galerkin reduction with cantilever modes and Struble/Laplace decoupling yields the modal system solved here by direct integration (n=3).
3. At corrected steel stiffness ($E=2.10924\times10^{11}$ N/m²), increasing N_0, Gp, K_0 in the thesis ranges suppresses response only modestly (≤~1.5%). The direction matches the thesis; the magnitude does not, because the thesis used $E=10^9$.
4. Moving-mass vs moving-force: peak difference ~0.6% at baseline, but phase shift is systematic. Force model is not universally conservative for timing-sensitive checks.
5. Boundary conditions matter: cantilever free-end dominance means results cannot be transferred from simply-supported or clamped–clamped cases. Separate publication is justified.
6. Future work: sweep c (10–80 m/s) for resonance, K_0 up to 10^6 N/m², and include damping and non-uniformity strength ε as explicit parameters.

Declarations

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